BOUND BY
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Lectures Notes at I.A.S

(1) Numerical Maths for 5th T.S.O's

(2) Fluid Mech. for 5th T.S.O's

(3) " 6th T.S.O's

(4) Lighting Systems " starting on 31/1/58.

(5) Do - Do - for 7th T.S.O's starting from 23/3/59.

Notes from Several books for the C.S.I.R. Scheme, 1968 (July onwards)

* Give example of this

$$x^3 + 2x - 4 = 0$$

$$\left\{2\left(\sqrt{29/27} + 1\right)\right\}^{1/3} - \left\{2\left(\sqrt{29/27} - 1\right)\right\}^{1/3}.$$

Lectures on numerical mathematics to B.Sc. 5th T.S.C.'s.

16/1/56 – 8.30 to 10.5 – 2 periods – Solution algebraic & transcendental equations

(a) Horner's Process – Algebraic eqns – possibilities about number of roots – real & complex – quadratic, cubic*, quartic, quintic etc – Rules re. transformation of roots – illustration of forming coefficients of $f(x+r)=0$ by taking a cubic:

$$ax^3+bx^2+cx+d=0$$

$$a(x+r)^3+b(x+r)^2+c(x+r)+d=0$$

~~$ax^3+x^2(3ar+b)+x(3$~~

$$ax^3+x^2(3ar+b)+x(3ar^2+2br+c)+(ar^3+br^2+cr+d)=0$$

Write down.

a	b	c	d
0 →	ar	$br+ar^2$	ar^3+br^2+cr
	$b+ar$	ar^2+br+c	ar^3+br^2+cr+d
	ar	$2ar^2+br$	
	$b+2ar$	$3ar^2+2br+c$	
	ar		
	$b+3ar$		

Explanation. If $f(x)$ is continuous from $x=a$ & $x=b$, and if $f(a)$ & $f(b)$ have opposite signs, then there is at least one real root between a & b –

Explanation of Horner's process by taking eqn

$$x^3-4x^2+5=0$$

– At the last stage after getting 1.38, getting the remaining decimal places the ratio of the last to the last but one coefficient — determining the other two roots. Here the fact is used that we can neglect the last coefficient. To see this divide eqn (4), Whittaker & Robinson by x. Getting

$$x^2 + 140x - 5326800 + \frac{10472000}{x} = 0$$

If x be very large, we can neglect last term in comparison with the remainder

ie the two large roots are given by

$$x^2 + 140x - 5326800 = 0$$

- Second example of $x^3 + 3x^2 - 12x - 10 = 0$. Here after three steps he gets the eqn $x^3 + 1119x^2 + 267387x - 54883 = 0$

from which he takes, since $x = 2.7\ldots$ and hence x^3 is small, the quadratic

$$x = \frac{54883}{267387} - \frac{1119}{267387}x^2$$

and substitutes for $x = 0.205$ getting a still better approximation

Examples

(1) Wallis's equation: $x^3 - 2x - 5 = 0$

If $f(x) = x^3 - 2x - 5$, $f(1) < 0$, $f(2) < 0$, $f(3) > 0$ ie at least one roots between 2 & 3.

	1	0	−2	−5
Divide by 2]		2	4	4
		2	2	−1
		2	8	
		4	10	
		2		
		6		

$$x^3 + 6x^2 + 10x - 1 = 0$$

Multiply by 10]: $x^3 + 60x^2 + 1000x - 1000 = 0$.

$x = 0$, L.H.S is $-ve$, $x = 1$, L.H.S is $+ve$ — Note that last two terms are

$x^3 + 600x^2 + 100{,}000x - 1{,}000{,}000 = 0.$

$x = 9, -^{ve}, \quad x = 10, +^{ve}$

	600	100000	−1000000
1	9	5481	949329
	609	105481	−50671
	9	5562	
	618	111043	
	9		
	629		

2.09

$x^3 + 629x^2 + 111043x - 51671 = 0$

$x^3 + 6290x^2 + 11104300x - 50671000 = 0.$

~~50671000~~ / ~~11104300~~) ~~516710 (5~~ ~~44417~~2 / ~~72538.~~ 111043) 606710 (5 / 555215 / 51495

2.095

Diminish by an 5 & get one more step.

Ex. 2. $x^3 - 9x^2 + 23x - 14 = 0$ (for class).

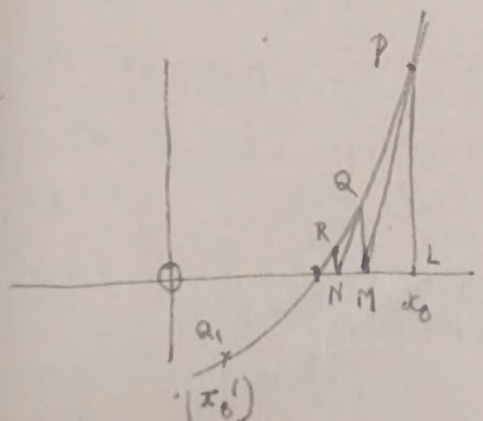
Ex. 3. $x^4 - 3x^2 + 75x - 10000 = 0$

b) Newton-Rhapson method. — Algebraic & transcendental eqns also:

eg (1) $\tanh x = 2x + \frac{1}{4}x^3$

(2) $3x - \cos x - 1 = 0.$

(3) $x^2 + 4\sin x = 0.$



$y = f(x).$ $OL = x_0$; $OM = x_0 - LM = x_0 - \frac{LM}{PL} \cdot PL$

$OM = x_1 = x_0 - \frac{PL}{PL/LM} = x_0 - \frac{f(x_0)}{f'(x_0)}$

$ON = x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$; $x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$

Conditions: (1) $f'(x) \neq 0$, $x_0' \leq x \leq x_0$

(2) $f''(x) \neq 0$ "

(3) must begin at begin at point where curve is convex to the origin

ie $f(x) f''(x) > 0$.

If $f(x) = 0$ only once between x_0' & x_0 and if (1) & (2) are satisfied, then start at the point where $f(x) f''(x) > 0$. ie $f(x)$ & $f''(x)$ have same sign

Slight modification: One can throughout use $f'(x_0)$ instead of $f'(x_r)$ in

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_0)}.$$

Another alternative: If α_1 be a first approximation, substitute $x = \alpha_1 + \delta$ in the eqn.

Solve it retaining only first power of δ getting $\delta = \delta_1$. 2^{nd} approx^n is $\alpha_2 = \alpha_1 + \delta_1$. Again

put $x = \alpha_2 + \delta$ and get δ_2, $\alpha_3 = \alpha_2 + \delta_2$ is 3^{rd} approx^n & so on.

Eg: $10x - x^3 = 3.1462644$

$x \simeq 0.3$, $x = 0.3 + \delta$, $3 + 10\delta - (.027 + .27\delta) = 3.1462644$

$9.73\delta = 0.17326$ ie $\delta = 0.017806$.

27
.09

Next approx is 0.3178. etc.

Application of this method to solution of simultaneous eqns

Eg: $x^3 + 2y^2 = 1$
$5y^3 + x^2 - 2xy = 4$

} from graph gets roots

$x = -0.6494$, $y = +0.7981$

Put $x = -0.6494 + \delta x$, $y = +0.7981 \delta y$

& gets two eqns for δx & δy & one solve alone gives result

Further notes on the Newton-Raphson method:

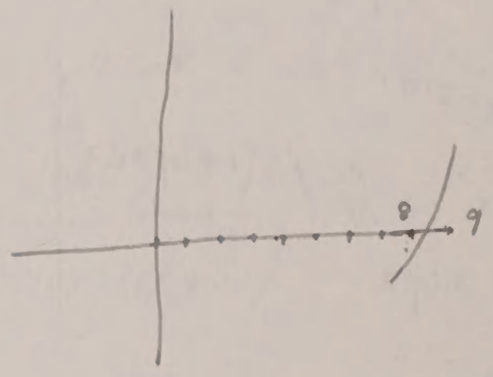
From Scarborough, p. 196-197

$$|E_n| \leq h_n^2$$

The number of reliable significant figures in h is equal to the number of zeroes between the decimal point and the first significant figure or

$x \log_e x = 18$ by the Newton-Raphson method

$$f(x) = x \log_e x - 18$$
$$f'(x) = \log_e x + 1.$$



$f(8) = 8(2.0794) - 18 = -ve$
$f(9) = 9(2.1972) - 18 = +ve.$
} ie root lies between 8 and 9.

First approx^n: $a_1 = 9 - \frac{f(9)}{f'(9)} = 9 - \frac{1.7748}{3.1972} = 9 - 0.5550 = 8.445.$

$$a_2 = 8.445 - \frac{f(8.445)}{f'(8.445)} = 8.445 - \frac{(8.445)(2.1336) - 18}{(2.1336 + 1)}$$

$$= 8.445 - \frac{0.02}{3.1336} = 8.445 - 0.006381.$$

$= 8.44500$
$\quad .00638$
$8.43862.$

8.4386

Whittaker gives answer as 8.46 – Could this be correct.

If $x = 8.43$, $f(x) = -0.03$ ($-ve$)
$x = 8.44$, $f(x) =$ very nearly 0.
ie root is between 8.43 & 8.44 & not 8.46 as in answer 2 W 4B

19.7748

.2492
.5048
.5556
1.7444

.5550

1336
.9266
.3992
1.2558

18.02

$\bar{2}.3010$
.4961
$\bar{3}.8049$

6381

8.43 (2.1318) − 18
8.44 ((2.1336) − 18
.9258 .9263
.3288 .3290
1.2546 1.2553

17.97

Ex: $x^x + 5x = 1000$.

$f(x) = x^x + 5x - 1000 = 0$. ; $f'(x) = x^x(1+\log x) + 5$.

$x = 3$, $f(3) = 27 + 15 - 1000 = -ve$

4, $f(4) = 256 + 20 - 1000 = -ve$

5, $f(5) = 3125 + 25 - 1000 = +ve$ } Root lies between 4 & 5

$y = x^x$; $\log y = x \log x$; $\frac{1}{y} \cdot y' = (1+\log x)\, x^x$

$a_1 = 5 - \frac{f(5)}{f'(5)} = 5 - \frac{2150}{3125(1+1.6094)+5}$

$= 5 - \frac{2150}{8159.375} = 5 - \frac{430}{1631.875}$

$= 5 - \frac{86}{326.375} = \underline{4.7365}$

$a_2 = 4.7365 - \frac{f(4.7365)}{f'(4.7365)} = 4.7365 - \frac{608.78}{4056}$

$= 4.7365 - 0.1501 = 4.4885$

$-0.1501 = 4.5864$

~~one more step appears necessary~~

one more step is necessary

~~In answer given $a_2 = 4.5$~~

According to the answer $f(4.5)$ & $f(4.6)$ should have diff. sign

multiply

$f(4.5) = (4.5)^{4.5} + 22.5 - 1000 = 869.8 + 22.5 - 1000 = -ve$ 4

$f(4.6) = (4.6)^{4.6}$ - " - " = [illegible] $- 1000 = +ve$

uses of effectiveness – Strategical kinematics –

Tactical analysis – Gunnery – functional expts – organisation.

(M) – Defn – types of problems – special problems – non-military.

(C) Defn – History – simple applns – Principles & theory – future.

(B) History – work of APM panel – Radar – guided missiles – American organisation

(T) – OFS – Amer. org – Navy applns – Chemical & physiological & psychological applns – Air applns – Repairing plans. (I.B.)

(Blackett) – Defn & remarks – convoy & depth charges –

(PRO) – History (8) IORSA

(6)

$Q = e(I_3 + 6/2 + 5/2)$

$I = \frac{1}{2} + 1 + 5/2$ $\quad S = 0$

$-e = -1$ $\quad \frac{1}{2}$ $\quad S = +1$

$I = \frac{1}{2} + \frac{5}{2}$ $\quad S = -1$

$-I = -\frac{1}{2} + \frac{5}{2}$

[illegible] kinsman's

[illegible] correct.

[illegible] not with 5 but with 4.6

$\frac{-1000}{1.5261) + 5}$

$6 - 0.05023$

4.54977
0.05023

$.5977.$

$.54977$

[illegible] $\therefore$ in h_2 there is only one zero after decimal

[illegible] of 2 variables) for the [illegible] of Calculus.

$\phi(x, y) = x + 3\log_{10} x - y^2$

$\psi(x, y) = 2x^2 - xy - 5x + 1.$

One of the point from is $(3.4\ 4\ 2.2)$. Here apply method of Multiple & Ref[illegible]

Ex: $x^x + 5x = 1000$.

$f(x) = x^x + 5x - 1000 = 0$. ; $f'(x) = x^x(1+\log x) + 5$.

$x = 3$, $f(3) = 27 + 15 - 1000 = -ve$

4, $f(4) = 256 + 20 - 1000 = -ve$

5, $f(5) = 3125 + 25 - 1000 = +ve$ } Root lies between 4 & 5

$y = x^x$; $\log y = x \log x$; $\frac{1}{y} \cdot y' = (1 + \log x) x^x$

$a_1 = 5 - \frac{f(5)}{f'(5)} = 5 - \frac{2150}{3125(1 + 1.6094) + 5}$

$= 5 - \frac{2150}{8159.375} = 5 - \frac{430}{1631.875}$

$= 5 - \frac{86}{326.375} = \underline{4.7365}$

$a_2 = 4.7365 - \frac{f(4.7365)}{f'(4.7365)} = 4.7365 - \frac{608.78}{4056}$

$= 4.7365 - 0.1501 = 4.4864$

one more step is necessary

Accurate to the answer $f(4.5)$ & $f(4.6)$ should have diff. sign

$.6754 \times 4.736 = 3.199$.

$1585(1 + 1.5552) + 5$

$(1585 \times 2.5552) + 5$

4051.

$f(4.5) = (4.5)^{4.5} + 22.5 - 1000 = 869.8 + 22.5 - 1000 = -^{ve}$

$f(4.6) = (4.6)^{4.6} + 23.0 - 1000 = 1119 + 23 - 1000 = +^{ve}$

$\cdot 6532 \times 4.5$	$\cdot 6628 \times 4.6$
32660	39768
26128	26512
2939.40	3048.88
2.9394	

ie Whittaker & Robinson's answer is correct.

The best thing is now to start not with 5 but with 4.6

$$a_1 = 4.6 - \left(\frac{f(4.6)}{f'(4.6)} = \frac{1119 + 23 - 1000}{1119\,(1 + 1.5261) + 5} \right)$$

$$= 4.6 - \frac{142}{2833.} = 4.6 - 0.05023$$

1119×2.5261

$= 4.54977$

4.54977
0.05023

.0489	2.1523
.4024	3.4513
.4513	2.7010

.05023 one more step required - yes. ∵ in h_2 there is only one zero after decimal

place & the first significant figure.

Example for simultaneous eqns: Scarborough (case of 2 variables) gives same method as in § 45, W & R, but more details of calculus.

Consider his example (Scarborough, p. 205)

$$\phi(x, y) = x + 3 \log_{10} x - y^2$$

$$\psi(x, y) = 2x^2 - xy - 5x + 1.$$

One of the points of intn is (3.4 & 2.2). Give effect method of Whittaker & Robinson

in $\log x$, $\log(x+\delta x)$ ie we have to use cents except ... Better use forms in Scarborough

& solve

$$\left.\begin{aligned} \phi(x_0,y_0) + h\left(\frac{\partial\phi}{\partial x}\right)_0 + k\left(\frac{\partial\phi}{\partial y}\right)_0 &= 0 \\ \psi(x_0,y_0) + h\left(\frac{\partial\psi}{\partial x}\right)_0 + k\left(\frac{\partial\psi}{\partial y}\right)_0 &= 0. \end{aligned}\right\}$$

Solving these we get $h_1 = 0.157$, $k_1 = 0.085$

Now take $x_1 = 3.4 + 0.157 = 3.557$
$y_1 = 2.2 + 0.085 = 2.285$ } 4 figures as before.

One more step gives the right answers. (Sc. p. 206).

Let us solve H & R, §46, by above method.

$$\phi(x,y) = x^3 + 2y^2 - 1$$

$$\psi(x,y) = 5y^3 + x^2 - 2xy - 4.$$

Here H & R take the starting values of (x_0, y_0) themselves as $\theta(-0.6494, +0.7981)$ — rather two arbitrary. Let us take $x_0 = -0.65$, $y_0 = +0.80$.

$$\frac{\partial\phi}{\partial x} = 3x^2, \quad \frac{\partial\phi}{\partial y} = 4y \qquad \left(\frac{\partial\phi}{\partial x}\right)_0 = 1.2675$$

$$\frac{\partial\psi}{\partial x} = 2x - 2y, \quad \frac{\partial\psi}{\partial y} = 15y^2 - 2x. \qquad \left(\frac{\partial\phi}{\partial y}\right)_0 = 3.20$$

$$\left(\frac{\partial\psi}{\partial x}\right)_0 = -2.9$$

$$\left(\frac{\partial\psi}{\partial y}\right)_0 = 10.9.$$

etc etc

.64

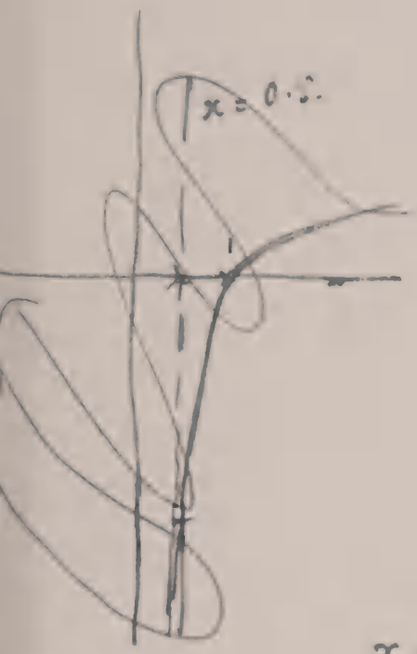
325
390
.4225

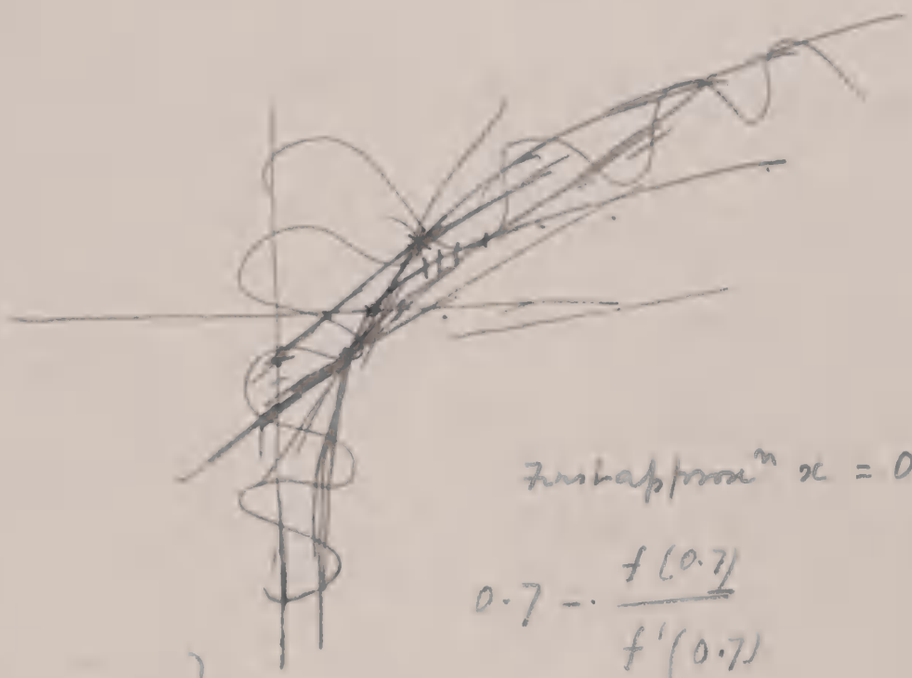
$-h^3$
-1.6
$9.60 + 1.3$
1.30

[illegible].

Ex: $x + \log_{10} x = 0.5$ correct to five places of decimals

$f(x) = x + \log_{10} x - 0.5$; $f'(x) = 1 + \frac{0.4343}{x}$





1st approx^n $x = 0.7$

$0.7 - \frac{f(0.7)}{f'(0.7)}$

$x = 0.7$:

T. 8451
T. 5000
T:3451
.7000
.0451

etc etc:

$= \log x$
$= 0.5 - x$
$+ y = 0.5$

Interpolation

8/1/56. General meaning, $f(x) \to \phi(x)$.

Weierstrass' theorems: about polynomials & ~~trigonometric~~ periodic functions:

Differences:

$\Delta' y_n = y_{n+1} - y_n$

$\Delta^2 y_n = \Delta' y_{n+1} - \Delta' y_n$

- - - - - - - - - -

$\Delta^r y_n = \Delta^{r-1} y_{n+1} - \Delta^{r-1} y_n$

Diagonal difference table

$\Delta_1 y_n = y_n - y_{n-1}$

$\Delta_2 y_n = \Delta_1 y_n - \Delta_1 y_{n-1}$

- - - - - - - - - -

$\Delta_r y_n = \Delta_{r-1} y_n - \Delta_{r-1} y_{n-1}$

Horl. Diffce table

$\Delta_m y_n = \Delta^m y_{n-m}$

$\Delta_1 y_n = \Delta^1 y_{n-1}$

$\Delta_2 y_n = \Delta_1 y_n - \Delta_1 y_{n-1} = \Delta^1 y_{n-1} - \Delta^1 y_{n-2} = \Delta^2 y_{n-2}$

. - - - - - - - - - .

Proof by induction

Effect of error in tabular values – Diagram & hand. tables – Ex. Se. p. 58.

Differences of a polynomial $y = f(x) = ax^n + bx^{n-1} + \cdots + kx + l$.

Let $h = \Delta x = \text{const}$] $y + \Delta y = a(x+h)^n + \cdots$

$\Delta y = a\{(x+h)^n - x^n\} + \cdots + kh.$

$\Delta = anhx^{n-1} + b'x^{n-2} + \cdots + k'x + l'.$

$\Delta^2 y = an(n-1)h^2x^{n-2} + b''x^{n-3} + \cdots + k''x + l''$

. . . .

$\Delta^n y = an(n-1)(n-2)\cdots 1 \cdot h^n.$ const:

ie n^{th} diffes are constant when h is const & conversely (values of x taken in A.P. in the table)

Newton's formula for forward interpolation: $y = f(x)$ tabulated n values. Let $y = \phi(x)$ be the interpolated polynomial

Write $\phi(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2) + \cdots$

$\cdots + a_n(x-x_0)(x-x_1)(\ )\cdots(x-x_{n-1})$

a_0, a_1, a_2 – to be determined such that for $x = x_0, \cdots, \phi(x) = y_0 \cdots$

$y_0 = a_0$ or $a_0 = y_0$

Let $x_1 - x_0 = x_2 - x_1 = \cdots = h$

$y_1 = a_0 + a_1(x_1 - x_0) = y_0 + a_1 h$

$a_1 = \dfrac{\Delta y_0}{h}.$

$a_2 = \Delta^2 y_0 / 2h^2$

$a_n = \Delta^n y_0 / n! h^n$

$\phi(x) = y_0 + \Delta y_0 \left(\dfrac{x-x_0}{h}\right) + \dfrac{\Delta^2 y_0}{2!}\left(\dfrac{x-x_0}{h}\right)\left(\dfrac{x-x_1}{h}\right)$

$+ \cdots + \dfrac{\Delta^n y_0}{n!}\left(\dfrac{x-x_0}{h}\right)\left(\dfrac{x-x_1}{h}\right)\cdots\left(\dfrac{x-x_{n-1}}{h}\right)$

Putting $\frac{x-x_0}{h} = u$, $\frac{x-x_1}{h} = \frac{x-x_0+x_0-x_1}{h} = u-1$, $\frac{x-x_2}{h} = u-2$.

$$\frac{x-x_{n-1}}{h} = u-n+1$$

$$\phi(x) = f(u) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \cdots + \frac{u(u-1)\cdots(u-n+1)}{n!}\Delta^n y_0 + \cdots$$

Newton's forward interpolation formula. — interpolation at beginning

Newton's backward interpolation formula $\quad (x_1 \cdots x_n)\ (y_1 \cdots y_n)$

Write $\phi(x) = a_0 + a_1(x-x_n) + a_2(x-x_n)(x-x_{n-1}) + \cdots + a_n(x-x_n)\cdots(x-x_1)$.

$$\phi(x) = \phi(u) = y_n + u\Delta_1 y_n + \frac{u(u+1)}{2!}\Delta_2 y_n + \cdots + \frac{u(u+1)\cdots(u+n-1)}{n!}\Delta_n y_n$$

— Interpolation at the end

Examples.

———o———

Scarborough, p. 68

Ex 1.

20736						
28561	7825					
38416	9855	2030				
50625	12209	2354	324			ϵ
65540	14915	2706	352	28		
83521	17981	3066	360	8		-4ϵ
104976	21455	3474	408	48		6ϵ
130321	25345	3898	416	8		-4ϵ
160000	29679	4334	444	28		ϵ

observing $28+\epsilon = 8-4\epsilon = 48+6\epsilon$ is satisfied exactly by $\epsilon = -4$

ie in the 5th row 65540 should be corrected to 65536

Ex. 2.

			$\Delta_1 y$	$\Delta_2 y$	$\Delta_3 y$	
19°	$12'$	$22''.4$				
19	25	54.7	$0^\circ 13' 32''.3$			
19	39	7.3	$0^\circ 13' 12''.6$	$+ 49''.7$		
19	51	53.8	$0^\circ 12' 46''.5$	$+ 26''.1$	$+ 6''.4$	$+\epsilon$
20	4	31.9	$0^\circ 12' 38''.1$	$+ 8''.4$	$- 17''.7$	-3ϵ
20	16	43.5	$0^\circ 12' 11''.6$	$+ 26''.5$	$+ 18''.1$	$+3\epsilon$
20	28	34.3	$0^\circ 11' 50''.8$	$+ 20''.8$	$- 5''.7$	$-\epsilon$

$\epsilon = +6''$ is the approximate correction

to be made in the fourth row when $19^\circ 51' 53''.8$ should be changed to

$19^\circ 51' 59''.8$

Ex. 3.

	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$-10 + 8.0319195$				
	.0115814			
" 8.0435009		.0003009		
	.0112805		.0000155	
8.0547814		−.0002854		−18
	.0109951		.0000137	
8.0657763		−.0002717		−5
	.0107234		.0000132	
8.0764997		−.0002585		−10.
	.0104649		.0000122	
8.0869646		−.0002463		
	.0102186			
8.0971832				

$u = 0^\circ 1' 23'' \cdot \frac{23}{60}$

$$\log \sin 37'23'' = \log \sin 37' + \frac{23}{60}(0.0115814) + \frac{\frac{23}{60}\left(\frac{23}{60}-1\right)}{2}(-0.0003009)$$

$$+ \frac{\frac{23}{60}\left(\frac{23}{60}-1\right)\left(\frac{23}{60}-2\right)}{6}(0.0000155)$$

$$+ \frac{\frac{23}{60}\left(\frac{23}{60}-1\right)\left(\frac{23}{60}-2\right)\left(\frac{23}{60}-2\right)}{24}(-0.0000018)$$

Ex. 5: From horizontal diff. table : $u = \frac{42'13'' - 43'}{60''} = -\frac{47}{60}$.

use backward interpolation formula.

Ex. 4. Here the value chosen is between the third & fourth in the table. So we need not write about the first two entries at all.

257°	$14'$	$22''.1$			
			$6^\circ 20' 46''.5$		
				$4' 29''.5$	
263	35	8.6			$29''.5$
			$6^\circ 25' 16''.0$		
				$4' 59''.0$	
270	0	24.6			$28''.5$
			$6^\circ 30' 15''.0$		
				$5' 27''.5$	
276	30	39.6			
			$6^\circ 35' 42''.5$		
283	6	22.1			

$$u = \frac{8\frac{5}{6} - 0}{12} = \frac{53}{72} = 0.7361$$

$$y = (257^\circ 14' 22''.1) + (0.7361)(6^\circ 20' 46''.5) + \frac{(0.7361)(-0.2639)}{2}(4' 29''.5)$$

$$+ \frac{(0.7361)(-0.2639)(-1.2639)}{6}(29''.5)$$

$$= (257^\circ 14' 22''.1) + (4^\circ 40' 17''.6) - (26''.2) + (1.2'')$$

$$= 261^\circ 54' 14''.7.$$

Ex. 6. Here the last entries are not required at all

$$u = \frac{-197/60}{12} = -\frac{197}{720}$$ //.

Whittaker & Robinson.

4·17: Ex. 7.

$x = -3, -2, -1, 0, 1$

$y = 16, 7, 4, 1, -8$

-3	16			
		-9		
-2	7		6	
		-3		-6
-1	4		0	
		-3		
0	1			
		-9		
1	-8			

$u = \frac{x+3}{1}$.

$$y = 16 + u(-9) + \frac{u(u-1)}{2!}(6) + \frac{u(u-1)(u-2)}{3!}(-6) + \frac{u(u-1)(u-2)(u-3)}{4!}(16)$$

$$y = 16 - 9(x+3) + 3(x+3)(x+2) - (x+3)(x+2)(x+1) + \frac{2(x+3)(x+2)(x+1)x}{3}$$

$$= 16 - 9x + 3x^2 - x^3 + \tfrac{2}{3}x^4$$
$$-27 + 15x - 6x^2 + 4x^3$$
$$-6 - 11x + \tfrac{22}{3}x^2$$
$$+ 4x.$$

$$y = \frac{2x^4}{3} + 3x^3 + \frac{13x^2}{3} - x. - 17$$

omitting the last term.

~~$y = 16 - 9x + 3x^2 - x^3$~~

~~$-27 + 15x - 6x^2 + 4x^3$~~

~~$-11x$~~

$y = 16 - 9(x+3) + 3(x+2)(x+3)$
$- (x+1)(x+2)(x+3)$

$= 16 - 9x + 3x^2 - x^3$
$-27 + 15x - 6x^2$
$+18 - 11x.$
$-6.$

$-x^2 + 3x - 5x + 1$

1R-p.43 $(p)_q = (p+1)_{q+1} - (p)_{q+1}$ ——— (1)

$$R.H.S = \frac{(p+1)!}{(q+1)!(p-q)!} - \frac{p!}{(q+1)!(p-q-1)!} = \frac{(p+1)! - (p-q)p!}{(q+1)!(p-q)!}$$

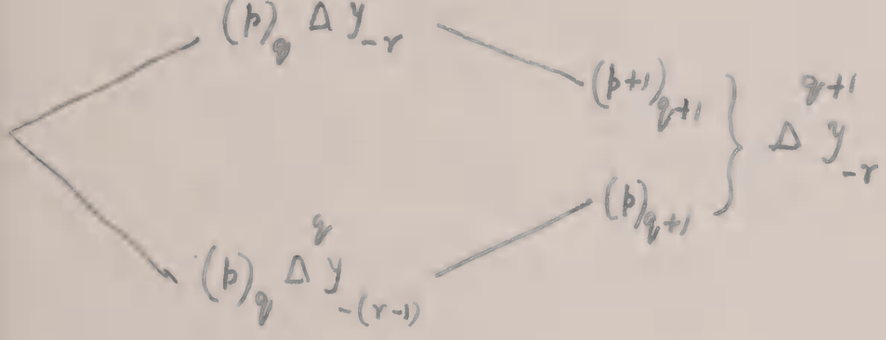
$$= \frac{p!}{(p-q)!\,q!} \cdot \frac{(p+1)-(p-q)}{q+1} = \frac{p!}{q!(p-q)!} = (p)_q$$

$$\Delta^q y_{-(r-1)} - \Delta^q y_{-r} = \Delta^{q+1} y_{-r} \quad \text{——— (2)}$$

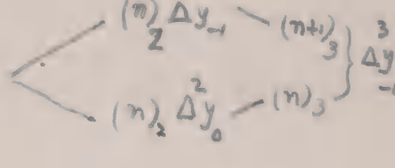
this is definition of $\Delta^{q+1} y_{-r}$

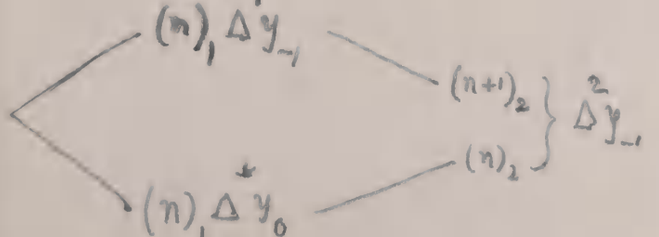
$$(p)_q \left\{ \Delta^q y_{-(r-1)} - \Delta^q y_{-r} \right\} = \left\{ (p+1)_{q+1} - (p)_{q+1} \right\} \Delta^{q+1} y_{-r}$$

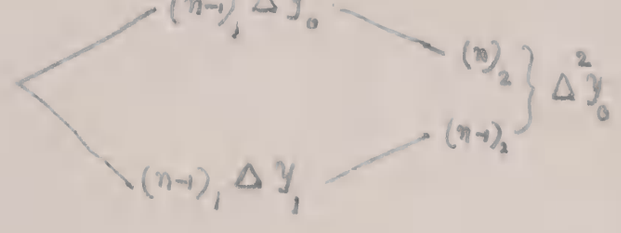
$$(p)_q \Delta^q y_{-r} + (p+1)_{q+1} \Delta^{q+1} y_{-r} = (p)_q \Delta^q y_{-(r-1)} + (p)_{q+1} \Delta^{q+1} y_{-r} \quad \text{——— (3)}$$



Sum along upper route = Sum along lower route in travelling from left hand vertex to right hand vertex of the Cayley diagram







W & R · p. 46.

Taking mean of (5) & (6) we have

$$y_n = y_0 + (n)_1 \cdot \frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{(n)_2 + (n+1)_2}{2} \Delta^2 y_{-1} + (n+1)_3 \cdot \frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2}$$

$$+ \frac{(n+1)_4 + (n+2)_4}{2} \cdot \Delta^4 y_{-2} + \cdots$$

$$= y_0 + n \cdot \frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{1}{2}\left\{ \frac{n!}{(n-2)!\,2!} + \frac{(n+1)!}{2!\,(n-1)!} \right\} \Delta^2 y_{-1} + \frac{(n+1)!}{3!\,(n-2)!} \cdot \frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} + \cdots$$

$$= y_0 + n \cdot \frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{n^2}{2} \cdot \Delta^2 y_{-1} + \frac{n(n^2-1)}{3!} \cdot \frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} + \cdots$$

which is the Stirling formula

Replacing n by u we get the general Stirling formula as in Scarborough p. 74

$\frac{1}{2}\left\{ \frac{n(n-1) + (n+1)n}{\cdot} \right\}$

$= \frac{1}{2} n \cdot 2n = \frac{n^2}{2}$

$\frac{(n+1)(n)(n-1)}{3!} = \frac{n(n^2-1)}{3!}$

Note that if in (III) we put

$u = -n, -(n-1), \cdots -1, 0, 1, 2, \cdots n$

we get the polynomial values $y_{-n}, \cdots y_0, \cdots y_n$

From the extended Lozenge diagram on p. 45 of W & R, we can write, starting from u_0 and going downwards from left to right along a diagonal

$$y_n = y_0 + (n)_1 \Delta y_0 + (n)_2 \Delta^2 y_0 + (n)_3 \Delta^3 y_0 + \cdots$$

$$= y_0 + n \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \cdots$$

which is the Newton–Gregory forward formula obtained by replacing y_n by y & n by u

In W & R - fig 3, p. 44, the reason for writing u_0 and u_1 in the left-hand centres of the upper & lower lozenges is the existence of the identity

$$y_0 + (n)_1 \Delta y_0 = y_1 + (n-1)_1 \Delta y_0 .$$

[$y_0 + n(y_1 - y_0) = y_1 + (n-1)(y_1 - y_0)$ obviously].

so that, consistent with the "Lozenge Rule" we can write u_0 & u_1.

Lagrange's Interpolation formula - Scarborough, p. 86.

$$\phi(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 + \cdots \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} \cdot y_n$$

is written as

$$= \sum_{i=0}^{n} \frac{\Pi_n(x)}{(x-x_i)\,\Pi_n'(x)} y_i \qquad \text{---------- (1)}$$

$$\left[\text{coeff of } y_i = \frac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)} . \right]$$

If $\Pi_n(x) = (x-x_0)(x-x_1)\cdots(x-x_n)$

$$\Pi_n'(x) = \sum_{i=0}^{n} \frac{\Pi_n(x)}{(x-x_i)}$$

[unnecessarily complicated although it looks apparently simple

(1) becomes $\sum_{i=0}^{n} \sum_{j=0}^{n} \frac{\Pi_n(x)}{(x-x_i)\,\Pi_n(x)} \frac{(x-x_j)}{} y_i = \sum_{i=0}^{n} \frac{y_i}{x-x_i} \{ (x-x_0) + (x-x_1) + \cdots \}$

$$= \frac{y_0}{x-x_0} \{ (x-x_0) + \} + \frac{y_1}{x-x_1} \{ (x-x_0) + \cdots \}$$

(1) becomes $\sum_{i=0}^{n} \frac{\Pi_n x}{(x-x_i) \sum_{j=0}^{n} \Pi_n(x)/(x-x_j)} y_i$

$$= \sum_{i=0}^{n} \frac{1}{(x-x_i) \sum_{j=0}^{n} 1/(x-x_j)} = \sum_{i=0}^{n} \frac{y_i}{(x-x_i)\left\{ \frac{1}{x-x_0} + \frac{1}{x-x_1} + \cdots \frac{1}{x-x_n} \right\}}$$

$$= \sum \frac{(x-x_0)(x-x_1) - (x-x_0)}{(x-x_i) \sum (x-x_0) - (x-x_n) + -} \Big\} \text{ etc.}$$

$$\phi(x) = \sum_{i=0}^{n} \frac{(x-x_0)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)} \cdot y_i$$

(1) is wrongly written in Scarborough $\Pi_n'(x)$ should be $\Pi_n'(x_i)$ — fool!

Inverse interpolation – Lagrange's formula $\psi(y) = x_0 \frac{(\ )\cdots(\ )}{(\ )\cdots(\ )} + \cdots$

Reversion of series – for eg of Newton–Gregory formula, etc.

Reversion of $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$ ——— (1)

$x = \frac{y-a_0}{a_1}$ ~~$y = a_0 + a_1 x + a_2 x^2$~~

Let $x = \left(\frac{y-a_0}{a_1}\right) + c_1\left(\frac{y-a_0}{a_1}\right)^2 + c_2\left(\frac{y-a_0}{a_1}\right)^3 + \cdots$ ——— (2)

Substituting (2) in (1)

$$y = a_0 + a_1\left\{\frac{y-a_0}{a_1} + c_1\left(\frac{y-a_0}{a_1}\right)^2 + c_2\left(\frac{y-a_0}{a_1}\right)^3 + \cdots\right\}$$
$$+ a_2\left\{\qquad\qquad\right\}^2 + a_3\left\{\qquad\qquad\right\}^3 + \cdots$$

Comparing coeffts: for y identically satisfied.

Coeff of y^2: $0 = a_1 c_1/a_1^2 + a_2/a_1^2$ i.e. $c_1 = -a_2/a_1$

Coeff of y^3: $0 = a_1 c_2/a_1^3$

$$c_1 + \frac{a_2}{a_1} = 0.$$

$$c_2 + \frac{a_2}{a_1}\cdot 2c_1 + \frac{a_3}{a_1} c_0$$

$$c_2 = -\frac{a_3}{a_1} + 2\cdot\left[a_2/a_1\right]^2$$

$$\frac{y-a_0}{a_1} = \frac{y-a_0}{a_1} + c_1\left(z\right)^2 + c_2 z^3 + \cdots$$

$$+ \frac{a_2}{a_1}\left(z + c_1 z^2 + \cdots\right)^2 + \frac{a_3}{a_1}\left(z + c_1 z^2 + \cdots\right)^3 + \cdots$$

$$0 = z^2\left\{c_1 + c_2 z + c_3 z^2 + \cdots\right) + \frac{a_2}{a_1} z^2\left(1 + c_1 z + c_2 z^2 + \cdots\right)^2 + \frac{a_3}{a_1} z^3\left(1 + c_1 z + c_2 z^2 + \cdots\right)^3 \cdots$$

$$\left(c_1 + c_2 z + c_3 z^2 + \cdots\right) + \frac{a_2}{a_1}\left(1 + c_1 z + c_2 z^2 + \cdots\right)^2 + \frac{a_3}{a_1} z\left(1 + c_1 z + c_2 z^2 + \cdots\right)^3 + \cdots = 0.$$

Coefficients of $z^0, z, z^2, \cdots$ etc.

Obsertion.

Problems for the Class:

Scarborough: p. 83, Ex. 1, 2, 4.
p. 95, Exc. 1, 2.
p. 172, Exc 2, ✱, 6, 7.

Whittaker & Robinson: p. 17, Ex. 9, 11,
p. 34, ex 6, 8.
p. 51, Exc. 3.
p. 69, Ex, 8, 9.
p. 162, Exc 1, 5, 6,

Milne. p. 85, Ex 1,
p. 130, Ex. 9, 10.

9th Eulerian - Maclaurin formula:

$$\frac{1}{h}\int_{x_0}^{x_0+nh} f(x)\,dx = \left(\frac{1}{2}y_0 + y_1 + y_2 + \cdots + \frac{1}{2}y_n\right)$$
$$- \frac{B_1 h}{2!}\left(y_n' - y_0'\right) + \frac{B_2 h^3}{4!}\left(y_n''' - y_0'''\right)$$
$$- \frac{B_3 h^5}{6!}\left(y_n^{v} - y_0^{v}\right) + \cdots$$

B_1, B_2, B_3 - are Bernoullis numbers defined

as $$\frac{1}{e^\theta - 1} = \frac{1}{\theta} - \frac{1}{2} + \frac{B_1}{2!}\theta - \frac{B_2}{4!}\theta^3 + \frac{B_3}{6!}\theta^5 - \cdots$$

$$B_1 = \frac{1}{6},\ B_2 = \frac{1}{30},\ B_3 = \frac{1}{42},\ \cdots$$

$$\frac{1}{e^\theta - 1} = \frac{1}{\left(\theta + \frac{\theta^2}{2!} + \cdots\right)} = \frac{1}{\theta\left(1 + \frac{\theta}{2!} + \cdots\right)}$$

$$= \frac{1}{\theta}\left(1 + \frac{\theta}{2!} + \cdots\right)^{-1}$$

$$= \frac{1}{\theta}\left\{1 - \frac{\theta}{2!} + \frac{\theta^2}{12} + \text{terms in } \theta^4 \text{ etc}\right\}$$

$$= \frac{1}{\theta} - \frac{1}{2} + \frac{\theta}{12} - \frac{\theta^3}{720} + \cdots \text{ etc.}$$

Applⁿ of Euler-Maclaurin formula

(i) Put $f(x) = x^p$, $x_0 = 0$, $h = 1$

$y_0 = 0,\ y_1 = 1^p,\ y_2 = 2^p,\ \ldots\ y_n = n^p$

$$\int_0^n x^p \cdot dx = \frac{n^{p+1}}{p+1} = \frac{1}{2}(0 + 1^p + 2^p + \cdots + (n-1)^p + \frac{1}{2}n^p)$$

$$\oplus\ -\frac{1}{12} p n^{p-1} + \frac{1}{720} p(p-1)(p-2) n^{p-3} - \cdots$$

$f'(x) = p x^{p-1}$

$f''(x) = p(p-1) x^{p-2}$

$f'''(x) = p(p-1)(p-2) x^{p-3}$

$f_n' = p n^{p-1}$, $f_0' = f_0''' = \cdots = 0$

$f_n''' = p(p-1)(p-2) n^{p-3}$

ie $1^p + 2^p + \cdots + n^p = \frac{n^{p+1}}{p+1} + \frac{n^p}{2} + \frac{1}{12} p n^{p-1} - \frac{1}{720} p(p-1)(p-2) + \cdots$

last term being $\frac{1}{2}$ or $\frac{1}{2}^2$

Ex: Prove that $S_7 + S_5 = 2 S_3^2$

$$S_7 = \frac{n^8}{8} + \frac{n^7}{2} + \frac{7n^6}{12} - \frac{7n^4}{24} + \frac{1}{12} n^2$$

$$S_5 = \frac{1}{6} n^6 + \frac{1}{2} n^5 + \frac{5}{12} n^4 - \frac{1}{12} n^2$$

etc.

7×6·5·4

~~720 360 240~~

~~1008 720~~

~~504 252~~

24

(ii) Stirling's approximation to $n!$

On Euler-Maclaurin formula with $h = 1$, $f(x) = \log x$.

$$\log(n!) = (n + \tfrac{1}{2}) \log n - n + \tfrac{1}{2} \log 2\pi + \frac{1}{12n} - \frac{1}{360 n^3} + \cdots$$

$$n! = n^{n+\frac{1}{2}} \sqrt{2\pi} \cdot e^{-n} \left(1 + \frac{1}{12n} + \cdots\right)$$

Ex: Evaluate $\log_{10}(79!)$

Ex-9 $\int_{x_0}^{x_4} y\,dx = \frac{4h}{9}(y_0 + 2y_1 + 3y_2 + 2y_3 + y_4)$ is exact for polynomial of degree 3.

ie Δ^4 etc - are all zero. Using (48.1) p. 138 of Scarborough.

$$I_0^4 = 2h\left[(y_1 + y_3) + \frac{1}{6}(\Delta^2 y_0 + \Delta^2 y_2 + \Delta^2 y_4)\right]$$

$$= 2h\left[(y_1 + y_3) + \frac{1}{6}(y_3 - 2y_1 + y_0 + y_4 - 2y_3 + y_2) + \frac{1}{6}\cdot y_4\right]$$

$$= \frac{h}{3}\left[6y_1 + 6y_3 + y_3 - 2y_1 + y_0 + y_4 - 2y_3 + y_2 + y_4\right]$$

$$= \frac{h}{3}\left[y_0 + 4y_1 + y_2 + 5y_3 + 2y_4\right]$$

not the right one

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$

$= (y_2 - y_1) - (y_1 - y_0)$

$= y_2 - 2y_1 + y_0$

$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$

$= (y_4 - y_3) - (y_3 - y_2)$

$= y_4 - 2y_3 + y_2$

$\Delta^2 y_4 = \Delta y_5 - \Delta y_4$

$= (y_6 - y_5) - (y_5 - y_4)$.

Ex. 10. $\int_{x_0}^{x_6} y\,dx = 3h(y_1 - y_2 + 3y_3 - y_4 + y_5)$ is exact for polynomial of degree 3.

Using Euler-Maclaurin formula with $y = ax^3 + bx^2 + cx + d$.

$y' = 3ax^2 + 2bx + c$, $y'' = 6ax + 2b$, $y''' = 6a$.

$y_4' - y_0' = 3a(x_4^2 - x_0^2) + 2b(x_4 - x_0)$.

$= 3a \cdot 16h^2 + 2b \cdot 4h$.

Ex. 9. Using (48.1) p. (45.1) p. 131 of Scarborough & putting $n = 4$ and neglecting all differences above the ~~third~~ fourth

$$\int_{x_0}^{x_4} y\,dx = h\left[4y_0 + 8\Delta y_0 + \left(\frac{64}{3} - 8\right)\frac{\Delta^2 y_0}{2} + \left(64 - 64 + 16\right)\frac{\Delta^3 y_0}{6}\right] +$$

$$= h\left[4y_0 + 8(y_1 - y_0) + \frac{20}{3}(y_2 - 2y_1 + y_0) + \frac{8}{3}(y_3 - 3y_2 + 3y_1 - y_0) + \frac{14}{45}(y_4 - 4y_3 + 6y_2 - 4y_1 + y_0) \right]$$

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$$

$$= (\Delta y_2 - \Delta y_1) - (\Delta y_1 - \Delta y_0)$$

$$= \Delta y_2 - 2\Delta y_1 + \Delta y_0$$

$$= (y_3 - y_2) - 2(y_2 - y_1) + (y_1 - y_0)$$

$$= y_3 - 3y_2 + 3y_1 - y_0$$

$$\Delta^4 y_0 = y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$$

$$\frac{h}{45}\left[\begin{array}{l} 180 y_0 \\ -360 y_0 + 360 y_1 \\ +300 y_0 - 600 y_1 + 300 y_2 \\ -120 y_0 + 360 y_1 - 360 y_2 + 120 y_3 \\ +14 y_0 - 56 y_1 + 84 y_2 - 56 y_3 + 14 y_4 \end{array} \right]$$

$$= \frac{h}{45}\left[14 y_0 + 64 y_1 + 24 y_2 - 64 y_3 + 14 y_4 \right]$$

new algorithm needed

$$\frac{256}{2} \quad 11 \times 64 \quad 704$$

$$\frac{1024}{5} - 384 + \frac{704}{3} - 48$$

$$3072 - 5760 + 3520 - 720$$

$$\frac{14}{15} \cdot \Delta^4 y_0$$

$$\frac{6592}{6480}$$
$$112$$

Sat 6/2/58 – Examples.

(1)

(A) $$\left\{ \begin{array}{l} \dfrac{dx}{dt} - 2\dfrac{dy}{dt} + x = \sin t \\ \dfrac{d^2x}{dt^2} - \dfrac{dy}{dt} + 2x - y = \log \cos t \end{array} \right\}$$ given $t = 0$, $x = 1$, $y = 2$, $\dfrac{dx}{dt} = 0$.

or $x_0 = 1$, $y_0 = 2$, $\dot{x}_0 = 0$. (3 arbitrary constants)

$$\left\{ \begin{array}{l} \dot{y} = \frac{1}{2}(x + \dot{x} - \sin t) \\ \ddot{x} = y - 2x + \dot{y} + \log \cos t \end{array} \right\}$$ (B)

(C) $$\left\{ \begin{array}{l} x = x_0 + \dot{x}_0 t + \dfrac{\ddot{x}_0 t^2}{2!} + \dfrac{\dddot{x}_0 t^3}{3!} + \dots \\ y = y_0 + \dot{y}_0 t + \dots \end{array} \right\}$$ $$\left\{ \begin{array}{l} \dddot{y} = \frac{1}{2}(\dot{x} + \ddot{x} - \cos t), \quad \ddot{y} = \frac{1}{2}(\ddot{x} + \dddot{x} + \sin t) \\ \dddot{x} = \dot{y} - 2\dot{x} + \ddot{y} - \tan t; \quad x^{IV} = \ddot{y} - 2\ddot{x} + \dddot{y} - \sec^2 t \end{array} \right\}$$

$y_0 = \frac{1}{2}$ .. $\dot{x}_0 = 0 \rightarrow \dot{y}_0 = \frac{1}{2} \rightarrow \ddot{x}_0 = \frac{1}{2} \rightarrow \ddot{y}_0 = -\frac{1}{4} \rightarrow \dddot{x}_0 = \frac{1}{4}$ etc alternately from

Substituting these in (C) $x = 1 + \frac{1}{4}t^2 + \frac{1}{24}t^3 - \frac{5}{64}t^4 - \cdots$

$y = 2 + \frac{1}{2}t - \frac{1}{8}t^2 + \frac{1}{16}t^3 - \cdots$

and $\dot{x} = \frac{1}{2}t^2 + \frac{1}{8}t^2 - \frac{5}{16}t^3 - \cdots$ } (D)

Putting $t = -0.10, -0.05, 0.05, 0.10$ in (D) we get along with initial values five values for $x, y, \dot{x}$ in the first four lines, and then for $\dot{y}$ & $\ddot{x}$ from (B).

For starting a new line, we use the formula:

$$\Delta \dot{x} = h\left(\ddot{x} + \frac{1}{2}\Delta_1 \ddot{x} + \frac{5}{12}\Delta_2 \ddot{x} + \frac{3}{8}\Delta_3 \ddot{x} + \frac{1}{3}\Delta_4 \ddot{x}\right)$$

and for finding Δx in the new line: (in the same line as the new $\dot{x}$)

$\Delta x = h\left(\dot{x} - \frac{1}{2}\Delta_1 \dot{x} - \frac{1}{12}\Delta_2 \dot{x} - \frac{1}{24}\Delta_3 \dot{x} - \frac{1}{38}\Delta_4 \dot{x}\right)$ — from (66.2)

(since $\dot{x}$ is the one found above)

and next find $\dot{y}$ & $\ddot{x}$ from (B) [If increase in x i.e. Δx is to be found from $\dot{x}$, etc in the new line, see 66.2]

then $\Delta \dot{x} = h\left(\ddot{x} - \frac{1}{2}\Delta_1 \ddot{x} - \frac{1}{12}\Delta_2 \ddot{x} - \frac{1}{24}\Delta_3 \ddot{x} - \frac{1}{38}\Delta_4 \ddot{x}\right)$

to cheque $\Delta \dot{x}$ of the new line.

Consider table on p. 273. Let us get the sixth line from the first five

(a) $\Delta \dot{x} = 0.05\left[0.5156 + \frac{1}{2}(0.0054) + \frac{5}{12}(-0.0048) + \frac{3}{8}(-0.0002) + \frac{1}{3}(-0.0003)\right]$

$= 0.05 \begin{bmatrix} 0.5156 & -0.0010 \\ 0.0027 & -0.0001 \\ & -0.0001 \end{bmatrix} = 0.0258$

.5183
.0012
5171
-.0855

$\dot{x} = 0.0509$
$+ 0.0258$
$.0767$ Correct

$$\Delta x = 0.05\left[0.0767 - \frac{1}{2}(0.0258) - \frac{1}{12}(0.0002) + \frac{1}{24}(0.0001) - \frac{1}{38}(0.0003)\right.$$

$$= 0.05\left[\begin{matrix}0.0767 - 0.0129 \\ -0.0002 \\ -0.0001\end{matrix}\right.$$

.0767
.0132
.0635 | 1.0025
.0032
.003175 | 1.0057 Correct

$$\dot{y} = \frac{1}{2}(x + \dot{x} - \sin t)$$

$$= \frac{1}{2}\left[1.0057 + 0.0767 - \sin(0.1500)\right]$$

$= \frac{1}{2}[1.0057 + 0.0767 - 0.1500]$ above

$= 0.4662$ but using correct value

we get correct value.

1.0824
.1500
.9324

Next find $\ddot{x}$.

$\ddot{x}$ number largest:

Then check for $\Delta\dot{x} = h\left(\ddot{x} - \frac{1}{2}\Delta\ddot{x} - \cdots\right)$.

Ex.2: $\frac{d^2y}{dx^2} - \sin y + 1 = 0$. or $y'' = \sin y - 1$

given $y = 0.1132$ and $y' = 0$ when $x = 0$.

$$\left[y = y_0 + y_0' x + \frac{y_0''}{2!}x^2 + y_0''' \frac{x^3}{3!} + y_0^{iv}\frac{x^4}{4!} + \cdots\right.$$ Taylor's series

$y_0 = 0.1$

$y''' = \cos y \cdot y'$

$y^{iv} = -\sin y \cdot y'^2 + \cos y \cdot y''$

$y^{v} = -\cos y \cdot y'^3 + 2y'\sin y \cdot y'' - \sin y \cdot y'y'' + y'''\cos y$

$= -\cos y \cdot y'^3 + \sin y \cdot y'y'' + y'''\cos y.$

$y_0''' = 0$, $y_0'' = 1$, $y_0' = 0$ $\qquad y_0^{iv} = \cos(0.1132)(-0.8870)$

$= -0.8874$

$y_0'' = \sin(0.1132) - 1 = -0.88704$;

$y^v = 0.$

$y = 0.1132 - 0.4435x^2 - 0.0367x^4$.

$x = -0.10, -0.05, 0.05, 0.10$ give alongwith y_0, 5 values of y.

Then tabulate x, y and y'' as given by the diff. eqn & also tabulate differences of y_n''

$$y_{n+1} = 2y_n - y_{n-1} + h^2\left[y_n'' + \frac{1}{12}(\Delta_2 y_n'' + \Delta_3 y_n'' + \Delta_4 y_n'')\right] \quad \text{—(1)}$$

$$y_n = 2y_{n-1} - y_{n-2} + h^2\left(y_n'' - \Delta_1 y_n'' + \frac{1}{12}\Delta_2 y_n''\right) \quad \text{—(2)}$$

The second formula is used to check the new line obtained from the first formula

In this case we find y_6 from the first five lines from (1). Then the y_n on the left hand side of (2) which is to be checked is this new y ie y_6 itself ie we use in (2) y_6'' etc. from the new line whereas in (1) we used y_5'', etc.

Ex. 3. $\frac{d^2\theta}{dt^2} + 0.2\frac{d\theta}{dt} + 10\sin\theta = 0$.

$\frac{d\theta}{dt} = \dot\theta$, $\frac{d\dot\theta}{dt} + 0.2\dot\theta + 10\sin\theta = 0$. Given for $t = 0$, $\theta_0 = 0.3$, $\dot\theta_0 = 0$

i.e. $\ddot\theta_0 = -10\sin(0.3) = -2.9552$.

or $\ddot\theta = -0.2\dot\theta - 10\sin\theta$.

$\theta = \theta_0 + \dot\theta_0 t + \ddot\theta_0\frac{t^2}{2!} + \dddot\theta_0\frac{t^3}{3!} + \cdots$ by Taylor's expansion

$\dddot\theta = -0.2\ddot\theta - 10\cos\theta\cdot\dot\theta$

$\theta^{iv} = -0.2\dddot\theta + 10\sin\theta\cdot\dot\theta^2 - 10\cos\theta\cdot\ddot\theta$

$\theta^{iv} = -0.2\theta^{iv} + 10\cos\theta\cdot\dot\theta^3 + 20\sin\theta\cdot\dot\theta\ddot\theta + 10\sin\theta\cdot\dot\theta\ddot\theta - 10\cos\theta\cdot\dddot\theta$

$= -0.2\theta^{iv} + 10\cos\theta\cdot\dot\theta^3 + 30\sin\theta\cdot\dot\theta\ddot\theta - 10\cos\theta\cdot\dddot\theta$, ..

$\dddot\theta_0 = (-0.2)\times(-2.9552) = 0.59104$

$\theta_0^{iv} = (-0.2)(-0.2)(-2.9552) - 10\cos(0.3)(-2.9552) = 28.1139$.

$\theta_0^{v} = -11.2692$, $\theta_0^{vi} = -188.903$.

$$\theta = 0.3 - \frac{2.9552}{2}t^2 + \frac{0.59104\,t^3}{6} + \cdots$$

$$\dot\theta = -2.9552t + \frac{0.59104\,t^2}{2} + \cdots$$

Putting $t = -\frac{1}{10}, -\frac{1}{20}, 0, \frac{1}{20}, \frac{1}{10}$ we get θ & corresponding $\dot\theta$ & corresponding $\ddot\theta$ from eqn.

Then from $\Delta\dot\theta = \frac{1}{20}\left(\ddot\theta_n + \frac{1}{2}\Delta_1\ddot\theta_n + \frac{5}{12}\Delta_2\ddot\theta_n + \cdots\right)$

from new right line.

Then check this by using

$$\Delta\dot\theta = \frac{1}{20}\left(\ddot\theta_n - \frac{1}{2}\Delta_1\ddot\theta_n + \cdots\right)$$ Here $\ddot\theta_n$ is $\ddot\theta_6$ etc.

Then find θ in the new line use from

$$\Delta\theta = \frac{1}{20}\left(\dot\theta_n - \frac{1}{2}\Delta_1\dot\theta_n + \cdots\right)$$ where again $\dot\theta_n$ is $\dot\theta_6$

with $\dot\theta$ & θ in the new line add $\ddot\theta$ also from the right eqn.

Then check $\dot\theta$ by ↑ and if necessary correct it, Correct also θ and also $\ddot\theta$. Again repeat checking $\dot\theta$, θ & $\ddot\theta$ until new line is complete.

(Q) · p. 260 of Scarborough

Ex. 4: $\frac{dy}{dx} = 2x - y$. given $x_0 = 1$, $y_0 = 3$.

$$y = y_0 + xy_0' + \frac{x^2}{2!}y_0'' + \frac{x^3}{3!}y_0''' + \frac{x^4}{4!}y_0^{iv} + \cdots$$

$y_0' = -1$
$y_0'' = 1$
$y_0''' = -3$
$y_0^{iv} = 3$

$x = 3 - 2$

$$y = 3 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$$

$y' = 2x - y$
$y'' = 2 - y'$
$y''' = -y''$
$y^{iv} = -y'''$

Put $x = -0.10, -0.05, 0, 0.05, 0.10$ & find five values of y and then corresponding values of y' and tabulate $\Delta_2 y', \Delta_2 y'$

then to add new line $\Delta y = 0.05 \left[y_n' + \frac{1}{2}\Delta_1 y_n' + \frac{5}{12}\Delta_2 y_n' + \cdots \right]$

and to check $\Delta y = 0.05 \left[y_n' - \frac{1}{2}\Delta_1 y_n' - \frac{1}{12}\Delta_2 y_n' - \cdots \right]$.

Ex. 5. Find five consecutive starting values for the eqn

$$\frac{dy}{dx} = \frac{xy}{x^2+y^2} \quad \text{given } x_0 = 0,\ y_0 = 1. \qquad \text{(A)}$$

$$y' = \frac{xy}{x^2+y^2}; \quad y'' = \frac{(x^2+y^2)(xy'+y) - xy(2x+2yy')}{(x^2+y^2)^2}$$

$$\therefore y_0' = 0, \quad y_0'' = \frac{1}{1} = 1$$

Taking $h = 0.1$, $y_1' = y_0' + h y_0''$, & $y_{-1}' = y_0' - h y_0''$ } as first approx^n

$= +0.1$, $\quad y_{-1}' = -0.1$.

From Milne's formulae: $y_1 = y_0 + \frac{h}{24}(y_{-1}' + 16 y_0' + 7 y_1') + \frac{y_0'' h^2}{4}$

$y_{-1} = y_0 - \frac{h}{24}(7 y_{-1}' + 16 y_0' + y_1') + y_0'' h^2/4$ } (B)

Here give $y_1^{(1)} = y_{-1}^{(1)} = 1.0050$.

Substituting in (A) we get $y_1'^{(2)}$ & $y_{-1}'^{(2)}$. Again from (B) we derive

$y_1^{(2)}$ & $y_{-1}^{(2)}$ & again in (A) gives $y_1'^{(3)}$ & $y_{-1}'^{(3)}$

These are found to be same as $y_1'^{(2)}$ & $y_{-1}'^{(2)}$

Hence $y_1^{(2)}$ & $y_{-1}^{(2)}$ are taken correct values.

If we have found three values of y_{-1} y_0 y_1

Next find y_2 & y_{-2} from

$$\left.\begin{aligned} y_2 &= y_0 + \frac{2h}{3}(5y_1' - y_0' - y_{-1}') - 2y_0''h^2 \\ y_{-2} &= y_0 - \frac{2h}{3}(5y_{-1}' - y_0' - y_1') - 2y_0''h^2 \end{aligned}\right\} \quad (C)$$

Next check y_2 & y_{-2} from

$$y_{n+1} = y_{n-1} + \frac{h}{3}(y_{n-1}' + 4y_n' + y_{n+1}') \quad (D)$$

after finding y_2' & y_{-2}' from the diff. eqn. for the purpose of checking y_2 write (D) as

$$y_2 = y_0 + \frac{h}{3}(y_0' + 4y_1' + y_2')$$

& to check y_{-2} as $y_0 = y_{-2} + \frac{h}{3}(y_{-2}' + 4y_{-1}' + y_0')$

or $y_{-2} = y_0 - \frac{h}{3}(y_{-2}' + 4y_{-1}' + y_0')$

Ex. 6 – Ex. 2, p. 171 of Scarborough

To find for what value of x is y a minimum. Obviously between 0.2 & 0.5

x	y					
0.2	0.9181	−0.0207	+.0105			
0.3	0.8975	−0.0102	+.0091	−.0014	+.0007	−.0006
(y_0) 0.4	0.8873	−0.0011	+.0084	−.0007	+.0001	
0.5	0.8862	+0.0073	+.0078	−.0006		
0.6	0.8935	+0.0151				
0.7	0.9086					

Stirling

$$\frac{dy}{dx} = \frac{1}{0.1}\left[\frac{-0.0102 - 0.0011}{2} + u(.0091) + \frac{3u^2-1}{3!}\left(\frac{-.0014 - .0007}{2}\right) + \frac{4u^3 - 2u}{4!}(-.0006)\right]$$

$\frac{.0021}{12}$ .00018

.8862
.0007
.8869
.0015
.8854

Putting $\frac{dy}{dx}=0$ gives, neglecting last term $-\frac{0.0006}{4!}$ as being small, ⊕

$$-\frac{0.0113}{2}+0.0091u-(3u^2-1)(0.0002)=0$$

$$-.0006u^2+.0091u-.0056+.0002=0$$

$$0.0006u^2-0.0091u+0.0054=0$$ we want here the ~~positive~$ value of u.

$$6u^2-91u+54=0. \qquad u=\frac{91\pm\sqrt{91^2-1296}}{12}=\frac{91\pm 83.576}{12}$$

we want here x to lie between 0.4 & 0.5 ie $u<1$. Hence taking the negative sign, $u=\frac{7.424}{12}=0.6187$

ie $x=0.4+(0.1)(0.6187)$

$=0.4+0.06187=0.46187$

$$-.00054u^2+.0091u-.0056+.00018=0$$

$$5.4u^2-91u+54.2=0$$

ie $u=\frac{91-84.32}{12}=$ 6.68

Using Bessel's formula:

$$\frac{dy}{dx}=\frac{1}{h}\left[\Delta y_0+\frac{2u-1}{2}\cdot\frac{\Delta^2y_{-1}+\Delta^2y_0}{2}+\frac{3u^2-3u+\frac{1}{2}}{6}\Delta^3y_{-1}\right]$$

$$=\frac{1}{0.1}\left[.8862+\frac{2u-1}{2}\left(\frac{.0062}{2}\right)+\frac{3u^2-3u+\frac{1}{2}}{6}(.0084)\right]$$

Putting this equal to 0

$$.8862+\frac{2u-1}{2}(.0031)+\frac{6u^2-6u+1}{12}(.0084)=0.$$

$$.8862+u(.0031)-.00155+u^2(.0042)-u(.0042)+.0007=0$$

91

8281
1296

8 | 6985 (83.576
64
585
489
9600
8325
127500
116949
1055100

21.6 × 54.2

4328
864
1080
1170.72

8281
1171
7110 (84.32
64
710
656
5400
5049
35100
33724

$(u-\frac{1}{2})(u^2-u)$

$u^3-\frac{3}{2}u^2+\frac{1}{2}u$

$3u^2-3u+\frac{1}{2}$

.0042
.00231
11

$u^2(\cdot0042) - u(\cdot0011) + \cdot8834 = 0.$

$42u^2 - 11u +$ $\cdot0175$

$\cdot0087u = \cdot0033$

$u = \frac{33}{87}$) 330 (3793
261
690
609
810
783
270
264

$\Delta y_0 = \cdot0011$, $\frac{dy}{dx} = 0$ given

$$\cdot0011 + \frac{2u-1}{2}\left(\frac{\cdot0091+\cdot0084}{2}\right) + \frac{3u^2-3u+\frac{1}{2}}{6}(-\cdot0007) = 0.$$

$$\cdot0011 + \frac{2u-1}{2}(\cdot0087) + \frac{6u^2-6u+1}{12}(-\cdot0007) = 0.$$

$$\cdot0011 + (u-\tfrac{1}{2})(\cdot0087) - (u^2-u+\tfrac{1}{2})(\cdot00006)$$

$u = \cdot3793.$

$\cdot4 + 038$

$\cdot438$

~~Ex. 13, p. 173~~

Ex. 7 = Ex. 13, (14) p. 173 of Scarborough:

@ $\int_{\cdot5}^{1\cdot1} xy\,dx$.

$y_0 = 0 \cdot5 \times \cdot4804$ | $y_4 = \cdot9 \times \cdot7985$

$y_1 = \cdot6 \times \cdot5669$ | $y_5 = 1\cdot0 \times \cdot8658$

$y_2 = \cdot7 \times \cdot6490$ | $y_6 = 1\cdot1 \times \cdot9281.$

$y_3 = \cdot8 \times \cdot7262$

$$\text{Integral} = \frac{0\cdot1}{3}\Big[(\cdot5\times\cdot4804) + 4\{(\cdot6\times\cdot5669) + (\cdot8\times\cdot7262) + (1\cdot0\times\cdot8658)\} + 2\{(\cdot7\times\cdot6490) + (\cdot9\times\cdot7985)\} + (1\cdot1\times\cdot9281)\Big]$$

Ex. 8. Ex. 12, p. 173 of Scarborough:

$$I = \int_0^{\frac{1}{2}} \frac{x\,dx}{\cos x}.$$

Calculate and use (Ex 8·3) p. 141 of Scarborough.

x	y
0	
·1	
·2	
·3	
·4	
·5	

5.4.42

Ex. 9: Prove that $\int_{x_0}^{x_4} y\,dx = \frac{4h}{9}(y_0 + 2y_1 + 3y_2 + 2y_3 + y_4)$ is exact for polynomial of degree 3.

Let integral be $= A_0 y_0 + A_1 y_1 + A_2 y_2 + A_3 y_3 + A_4 y_4$. Also take $x_0 = a - 2h$, $x_1 = a - h$, $x_2 = a$, $x_3 = a + h$, $x_4 = a + 2h$.

By Simpson's rule, integral $= \frac{h}{3}\{y_0 + y_4 + 4y_1 + 4y_3 + 2y_2\}$ which is also exact for polynomial of degree 3. ie we should have

$$4(y_0 + 2y_1 + 3y_2 + 2y_3 + y_4) = 3(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$$

$$y_0 - 4y_1 + 6y_2 - 4y_3 + y_4 = 0. \text{ for a polynomial of degree 3.}$$

ie $(a-2h)^3 - 4(a-h)^3 + 6a^3 - 4(a+h)^3 + (a+2h)^3$

$$(y_4 - y_3) - 3(y_3 - y_2) + 3(y_2 - y_1) - (y_1 - y_0) = 0$$

$$\Delta y_3 - 3\Delta y_2 + 3\Delta y_1 - \Delta y_0 = 0.$$

$$(\Delta y_3 - \Delta y_2) - 2(\Delta y_2 - \Delta y_1) + (\Delta y_1 - \Delta y_0) = 0$$

$$\Delta^2 y_2 - 2\Delta^2 y_1 + \Delta^2 y_0 = 0$$

$$\Delta^3 y_1 - \Delta^3 y_0 = 0 \quad \text{ie } \Delta^4 y_0 = 0 \text{ true for a polynomial of degree 3}$$

Ex. 10.

Prove that $\int_{x_0}^{x_6} 3h(y_1 - y_2 + 2y_3 - y_4 + y_5)$ is exact for polynomial of degree 3.

ie $\frac{h}{3}(y_0 + y_6 + 4y_1 + 4y_3 + 4y_5 + 2y_2 + 2y_4) = \frac{9}{3}h(y_1 - y_2 + 2y_3 - y_4 + y_5)$

$$y_0 - 5y_1 + 11y_2 - 14y_3 + 11y_4 - 5y_5 + y_6 = 0$$

$$\Delta^4 y_0 - y_1 + 5y_2 - 10y_3 + 10y_4 - 5y_5 + y_6 = 0 \quad \text{ie } \Delta^6 y_0 - \Delta^5 y_1 = 0 \text{ which is true.}$$

Lectures on Fluid Mechanics to 5th T.S.O's

1st lecture: 12/3/56. (from V.R.T's notes)

1. Viscous fluids
2. Thin fluids
3. Gaseous fluids.

or

1. Highly viscous
2. Slightly viscous
   - thin liquids
   - gases.

Solids & fluids – change of shape – resistance to deformation – example of pitch (velocity of deformation being large or small) – force producing deformation → 0 as vel. of deformation → 0. as definition of fluid – on solids stress & strain – resistance to deformation while in motion – Experimental way of differentiating between 1 & 2 is to move a solid body in them for eg ~~oil~~ water syrup with same density – change of volume: diff. between thin liquids & gases – In cases where there is no appreciable alteration (small pressure change) of volumes gases & thin liquids obey same laws & are treated under hydro-& aero mechanics in narrow sense. Large change of volume brought about by high velocity comes under gas dynamics or Ballistics & brought about by pressure (very great height) comes under dynamic meteorology.

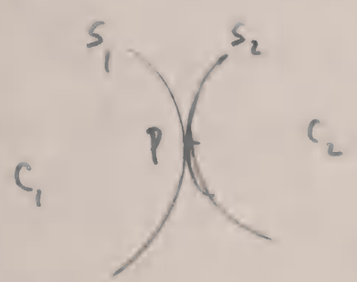
Fluids (F → 0 as v_{def} → 0 / (no resistance to change of shape ~~deformation~~ ie $\sigma(\frac{\partial v}{\partial x})$.

- Highly viscous fluids
- Slightly viscous
  - thin liquids
  - gases
    - little change of vol
    - Gas Dyn.
    - Aero
    - heat change of vol
      - Gas Dyn.
      - Aero

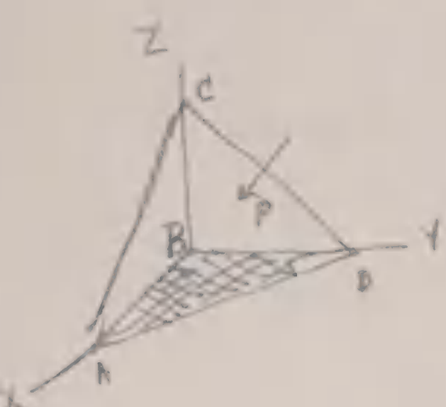
Hydro & aerodynamics

classical hydrodynamics & practical phenomena - Examples of pressure drop in pipes & resistance of a body moving through a fluid being zero (!) in old classical hydrodynamics - Emergence of hydraulics as a consequence by engineers - Difference: (1) analytic & synthetic (2) natural laws & natural phenomena (3) theory which was unreal and no underlying theory (4) Hydraulics a collection of unrelated problems & a science of coefficients - Modern hydrodynamics which grew as a result of growth of aeronautics & turbine engineering & a synthesis has been made between hydrodynamics & hydraulics - 4 steps in a physical theory - laminar & turbulent motion.

<u>Fluid pressure</u>:



C_1 & C_2 touching at P, dS being element of area of common tangent plane containing P. Force exerted by C_2 on C_1 across dS is independent of shapes of S_1 & S_2 as long as tangent plane remains the same. If this force be dF, then $\frac{dF}{dS}$ = stress at P exerted by C_2 on C_1 or by C_1 on C_2 in diff. dirn. This is in general oblique to dS - components normal & tangential are called <u>normal stress</u> & <u>shearing stress</u>. For a fluid in equilibrium there are no shearing stresses & normal stress assumed a <u>thrust</u> (not a tension) is called the <u>pressure</u>. To show pressure to be same for all orientations of dS, let ABC = dS & its projection on XOY = dS_2 - P & P_2 pressures on dS_2 & dS.



Component of Z = Body force/unit mass (like gravity etc) acting on ds in z-dirn

Let dS make angle α with XOY. Resolving forces on tetrahedral element along z-axis

$$p_2\,dS_2 - p\,dS\cos\alpha + \tfrac{1}{3}\,dS_2 \cdot PC \cdot \rho \cdot Z = 0.$$

$$p_2\,dS\cos\alpha - p\,dS\cos\alpha + \tfrac{1}{3}\,dS\cdot\cos\alpha \cdot PC\cdot\rho\cdot Z = 0$$

$$p_2 - p + \tfrac{1}{3}\rho\cdot PC\cdot Z = 0.$$

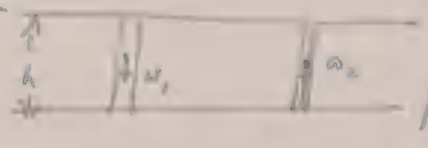
Going to limit as $PC \to 0$ $\quad p_2 = p$

Eqm of liquid under gravity.

$$p_1 = p_2$$

$$p_2 = p_1 + g\rho h \qquad \text{——(1)}$$

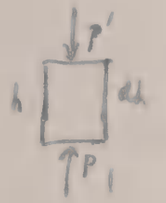
Het. liquid



At left $p_2 - p_1 = \omega_1 h$
At right $p_2 - p_1 = \omega_2 h$
$\therefore \omega_1 = \omega_2$
ie $\rho_1 = \rho_2$

For heterogeneous liquids, use eq (1) puts $\rho_1 = \rho_2$ for the same horizontal layer.

Eqm of gas under gravity: the atmosphere:



$$p'ds + g\rho h\,ds = p\,ds$$

$$p' - p = -g\rho\,dh$$

$$dp = -g\rho\,dh.$$

$$\frac{dp}{dh} = -g\rho \qquad (1)$$

perfect gas $p = R_g \rho T$ (2)

$$\frac{dp}{dh} = -\frac{p}{RT} \qquad (3)$$

(i) isothermal case: $RT = \text{const}$: $\frac{\rho}{\rho_0} = \frac{p}{p_0} = e^{-h/RT}$ (4) (5)

(ii) T decreases uniformly with increasing h ie $\frac{dT}{dh} = -\lambda$ (7)

$\lambda > 0$ is temp. gradient. $\Rightarrow T = T_0$ for $h = 0$

$$T = T_0 - \lambda h \qquad (8)$$

$$p = p_0 \left(1 - \frac{\lambda h}{T_0}\right)^{\frac{1}{\lambda R}} \qquad (9)$$

$$\frac{\rho}{p_0} = \frac{\rho T}{\rho_0 T_0}$$

ie $$\frac{\rho}{\rho_0} = \frac{p}{p_0}\left(\frac{T}{T_0}\right)^{-1} = \left(1 - \frac{\lambda h}{T_0}\right)^{\frac{1-\lambda R}{\lambda R}} \qquad (10)$$

(i) uniform atmosphere: $v = \text{const}$

$$pv = RT$$

$$\frac{dp}{dh} = -\frac{1}{v}\cdot$$

$$p = \frac{1}{v}\cdot H$$

ie $H = RT$.

$$p = p_0 e^{-h/RT} \qquad (10)$$

Solving (9) & (10)

$$h = \frac{T_0}{\lambda}\left[1 - \left(\frac{p}{p_0}\right)^{\lambda R}\right] = \frac{T_0}{\lambda}\left[1 - \left(\frac{\rho}{\rho_0}\right)^{\lambda R}\right] \quad (11)$$

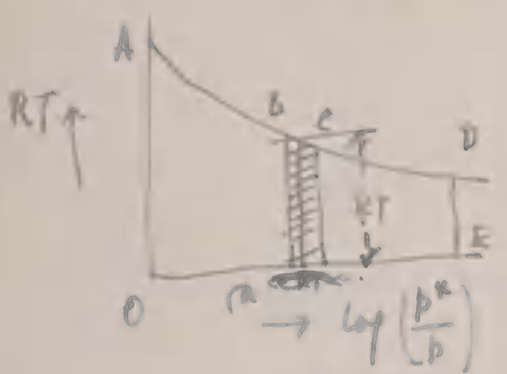
i.e. $$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{1-\lambda R}} \quad \text{or} \quad \frac{p}{\rho^\gamma} = \frac{p_0}{\rho_0^\gamma} \quad \text{where } \gamma = \frac{1}{1-\lambda R} \quad (12)$$

(12) is Poly tropic change of state.

(iii) no specific law of variation of T:

$$dh = -RT \cdot \frac{dp}{p}.$$

If p^* = pressure at $h = 0$, $dh = RT\, d\left(\log \frac{p^*}{p}\right)$ (14)



A graph shows that difference in level between B & C is the shaded area in general. Thus for any point D. The altitude is given by area OADE of A the point corresponding to standard $h = 0$.

Examples: p.6

Ex. 2. $dp = -g\rho\, dh$ Here we assume ρ = const = ρ_0

$p = Rg\rho T$.

$p = -g\rho_0 h + C$. i.e. $p - p_0 = -g\rho_0 h$. i.e. $C = p_0$

When $h = H$, $p = 0$. i.e. $C = g\rho_0 H$. $\therefore p_0 = g\rho_0 H$.

$$H = \frac{p_0}{g\rho_0} = RT.$$

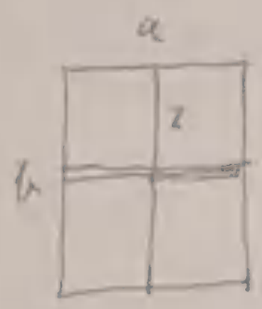
Ex. 3 & 4 are isothermal & adiabatic cases.

Ex. 5 for practical work.

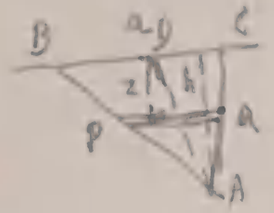
Exx. 9. 9

$$\bar{z} = \frac{\int_0^h z^2 \cdot a\,dz}{\int_0^h z \cdot a\,dz} = \frac{h^3/3}{h^2/2} = \frac{2}{3}h. \text{ on median by symm.}$$

(i)



(ii)



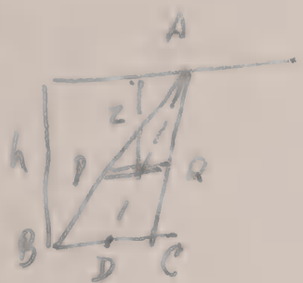
$$\bar{z} = \frac{\int_0^h z^2 \cdot PQ\,dz}{\int_0^h z \cdot PQ\,dz} = \frac{\int_0^h z^2 \cdot \frac{a}{h}(h-z)\,dz}{\int_0^h z \cdot \frac{a}{h}(h-z)\,dz}$$

$$= \frac{\int_0^h (hz^2 - z^3)\,dz}{\int_0^h (hz - z^2)\,dz} = \frac{\frac{h^4}{3} - \frac{h^4}{4}}{h^3/2 - h^3/3} = \frac{h^4/12}{h^3/6} = \frac{h}{2}$$

on median.

i.e. midpoint of AD

(iii)



As in (ii) but replace $\frac{a}{h}(h-z)$ by $\frac{a}{h} \cdot z$. & this gives

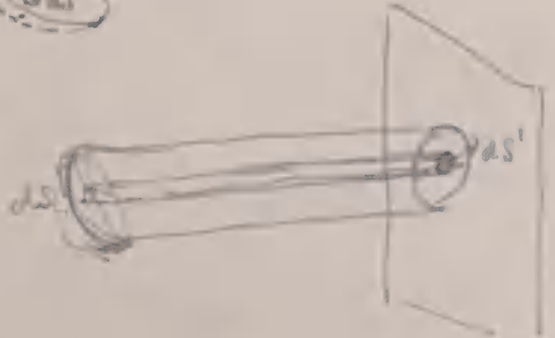
$$\bar{z} = \frac{\int_0^h z^3\,dz}{\int_0^h z^2\,dz} = \frac{h^4/4}{h^3/3} = \frac{3}{4}h. \text{ on AD}$$

Art. 7

vertical thrust on element dS = wt of superincumbent liquid

$\therefore$ Total vertical thrust = total wt of superincumbent liquid in the cylinder.

Horl. thrust: Project dS on a plane $\perp^r$ to the horl. direction considered. Horl. thrust on dS = horl. thrust on dS' since filament is in equilibrium. Hence thrust = thrust on the projected area at the C.P of the projection



For a closed surface, above construction shows that thrust zero & vertical thrust = diffce of wts of the superincumbent fluids on the two cylinders which equal wt of fluid occupying space of the closed surface & hence acts through C.G of this volume of fluid.

§8. Principle of Archimedes proved by what Prandtl calls principle of solidification (Prandtl, p. 2 & p. 12). - The last result of §7 is also obvious by principle of Archimedes

Buoyancy & centre of buoyancy - metacentre.

Examples. p. 11. (also get some more examples from Morris & Legge).

N & L. p. 215

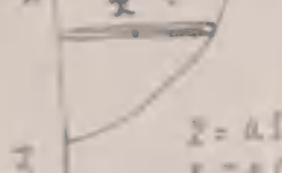
(1) A triangle is immersed in a homogeneous fluid, and its vertices are at depths a, b, c from the surface. Find depth of C.P.

Replacing by equimomental system of three particles with masses $\propto \frac{1}{3}A$ at midpoints at depths $\frac{1}{2}(b+c)$, $\frac{1}{2}(c+a)$, $\frac{1}{2}(a+b)$.

$$\bar{x} = \frac{\sum \frac{1}{2}(b+c)\cdot\frac{1}{2}(b+c)}{\frac{3}{4}\sum \frac{1}{2}(b+c)} = \frac{a^2+b^2+c^2+bc+ca+ab}{2(a+b+c)}$$

The two examples of triangular areas considered previously are particular cases of this obtained by putting (i) $b=c=0$. (ii) $a=0$ & $b=c$. giving results $\bar{x} = a/2$ & $\frac{3}{4}b$.

(2) Find the position of the C.P. of a quadrant of a circle in a vertical plane with the bounding radius in the free surface



$$\bar{x} = \frac{\sum p\,x\,ds}{\sum p\,ds}, \text{ etc.}$$

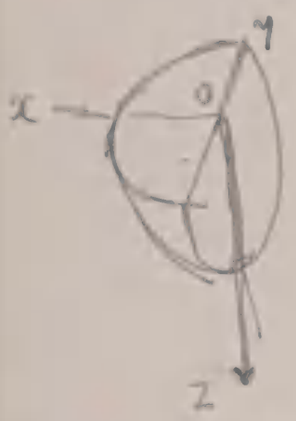
$$\bar{x} = \frac{\int_0^a x\frac{x}{2}\cdot x\,dz}{\int_0^a z\cdot x\,dz} = \frac{1}{2}\,\frac{\int_0^{\pi/2} a\sin\theta\cdot a^2\cos^2\theta\cdot a\cos\theta\,d\theta}{\int_0^{\pi/2} a\sin\theta\cdot a\cos\theta\cdot a\cos\theta\,d\theta} = \frac{a}{2}\,\frac{\int_0^{\pi/2}\cos^3\theta\sin\theta\,d\theta}{\int_0^{\pi/2}\cos\theta\sin\theta\,d\theta}$$

ie $\bar{x} = \frac{a}{2} \cdot \frac{\frac{2}{4} \cdot \frac{1}{2}}{\frac{1}{3} \cdot 1} = \frac{3a}{8}$.

$$\bar{z} = \int_0^{\pi/2} z \cdot z \cdot x\,dz \Big/ \int z \cdot x\,dz = a \cdot \frac{\int_0^{\pi/2} \sin^2\theta \cos^3\theta\,d\theta}{\int \sin\theta \cos^3\theta\,d\theta} = a \cdot \frac{\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}}{\frac{1}{3}} = \frac{3\pi a}{16}.$$

N 42. p. 231

(1) Find the pressure on the curved surface of a spherical quadrant with a plane face in the surface

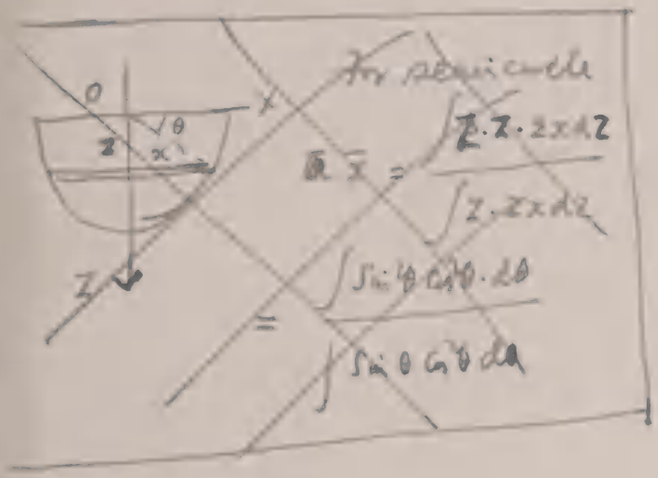


Pressure parallel to OX = pressure on the vertical face which is proj^n of curved surface

$$= g\rho \cdot \frac{4a}{3\pi} \cdot \frac{\pi a^2}{2} \qquad \left[\frac{4a}{3\pi} = \text{depth of C.G.}\right.$$

$$= \frac{2}{3} g\rho a^3.$$

and it acts at $\left(0, \frac{3\pi a}{16}\right)$. (vide above).



Pressure $\parallel$ to $Oy = 0$.

" Ox = wt. of fluid = $\frac{1}{3} g\rho\pi a^3$.

4 acts in the direction of the line $x = \frac{3}{8}a$

$\therefore$ ~~(illegible)~~ $\left(\frac{3}{8}a, 0, \frac{3\pi a}{16}\right)$ ~~(illegible)~~

The two forces meet at $\left(\frac{3}{8}a, 0, \frac{3\pi a}{16}\right)$ & hence

$$\text{resultant} = \frac{1}{3}\sqrt{\pi^2 + 4.9}\,\rho a^3$$

If θ = angle which force makes with x-axis, $\tan\theta = \frac{\frac{1}{3} g\rho\pi a^3}{\frac{2}{3} g\rho a^3} = \frac{1}{2}\pi$

Also for the point point. $\frac{z}{x} = \frac{3\pi a/16}{3a/8} = \frac{1}{2}\pi$

ie force passes thro' centre of sphere, which is obvious since pressures on all elements of curved surface are normal to it & hence pass through the centre.

N.A d, p. 237

(1) A cube floats in water with two faces horizontal. Discuss stability or instability of eqm

Let s = sp. gravity of the cube

$\therefore a^3 s = a^2 x$ or $x = as$

$$HM = \frac{Ak^2}{V} = \frac{a^2 \cdot a^2/12}{a^2 x} = \frac{a^2}{12x}$$

M.I of square about OX or

$k^2 = \frac{\Sigma p \cdot dx dy \cdot y^2}{\Sigma p \, dx dy}$

$= \frac{2a \cdot 2a^3/3}{2a \cdot 2a}$

$= 4a^2/3$.

or $a^2/12$ of a base length

Eqm is stable or unstable accg. as

$$HM \gtrless HG$$

$$\frac{a^2}{12x} \gtrless \frac{1}{2}(a-x)$$

ie $s^2 - s + \frac{1}{6} \lessgtr 0$

ie for stability s cannot lie between $\frac{1}{2} + \frac{\sqrt{3}}{6}$ & $\frac{1}{2} - \frac{\sqrt{3}}{6}$.

Between these values of s,

eqm is unstable

Experimental method of finding Metacentre

Let HG cut the deck at B. If wt W' (movable) be moved a distance c across the deck, $G \to G'$

$H \to H'$ & $H'G'$ cuts HG in M.

Taking moments about G'

$$W \cdot GM \cdot \sin\theta = W'(c\cos\theta + b\sin\theta)$$

$$GM = \frac{W'}{W}\left(\frac{c}{\theta} + b\right) \qquad [\tan\theta \cong \theta$$

$$= \frac{W' \cdot c}{W\theta} \qquad \left[\frac{c}{\theta} \text{ is large \& } b \text{ small}\right.$$

[b_1 = width of ship]

$MB = b$]

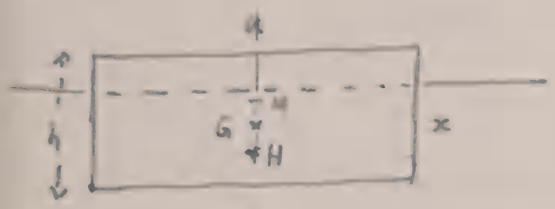
i.e. $GM = BB'$

and θ is given by the arc through which pendulum of length l moves : $\theta = \frac{s}{l}$.

$GM = \frac{N'Cl}{WS}$. which determines position of M.

P. 11.

(1)



$a^2 hS = a^2 x$ or $x = hs$.

$HM = \frac{Ak^2}{V} = \frac{a^2 \cdot a^2/12}{a^2 x} = \frac{a^2}{12x}$.

Stable if $HM \gtrless HG$. i.e. $\frac{a^2}{12x} \gtrless \frac{1}{2}(h-x)$.

$a^2 \gtrless 6x(h-x)$

$\gtrless 6hs(h-hs) \gtrless 6h^2 s(1-s)$.

$a^2 - 6h^2 s(1-s) \gtrless 0$.

$6h^2 s^2 - 6h^2 s + a^2 \gtrless 0$ $\quad h^2 s^2 - h^2 s + \frac{a^2}{6} \gtrless 0$.

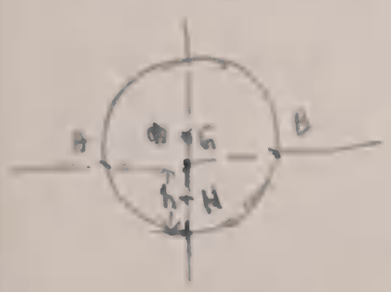
$s = \frac{h^2 \pm \sqrt{h^4 - \frac{4a^2 h^2}{6}}}{2h^2}$

$= \frac{h \pm \sqrt{h^2 - \frac{2a^2}{3}}}{2h}$

i.e. if s lies between $\frac{1 + \sqrt{1 - \frac{2a^2}{3h^2}}}{2}$ and $\frac{1 - \sqrt{1 - 2a^2/3h^2}}{2}$

Eqm is unstable.

(2)



Let AB = breadth of cross section = b. & l = length of cylinder

Then $HM = \frac{bl \cdot b^2/12}{V}$.

$\therefore$ Eqm is stable if $\frac{b^3 l}{12V} > HG$.

i.e. $b^3 l > 12V \cdot HG$

If $l > b$, then $b^3 l > b^4$. The result is therefore proved if we show that

$12V \cdot HG < b^4$.

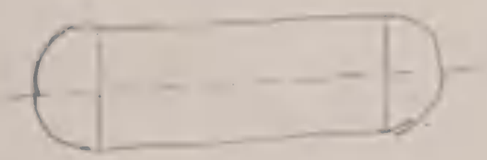
$V = \pi h^2(a - \frac{1}{3}h)$; $HG = \dfrac{(a-\frac{1}{2}h)^2}{a-\frac{1}{3}h}$. ; $\dfrac{b^2}{4} = a^2 - (a-h)^2$ ie $\dfrac{b^2}{8} = h(a-\frac{1}{2}h)$

$12V \cdot HG < \pi\, 12\pi h^2(a-\frac{1}{2}h)^2$; $b^4 = 64h^2(a-\frac{1}{2}h)^2$

obviously $12\pi < 64$ & hence the result.

3.

(i)



Thrust in horl. dirn = Thrust on the projection of the end on a vertical plane ⊥r to the axis

= Thrust on vertical circle of radius 3 ft

$= \pi a^2 \cdot g\rho a = \pi a^3 \rho \cdot g$.

(ii)

(a) on lower end, thrust = wt of superincumbent liquid.

$= (\frac{4}{3}\pi a^3 + \pi a^2 l)\rho g$.

on upper end, thrust $= g\rho(\pi a^3 - \frac{2}{3}\pi a^3) = \frac{1}{3}\pi a^3 g\rho$.

Other problems:

(1) Show that the resultant pressure on a plane area $= g\rho \bar{x} S$, where $\bar{x}$ is the depth of the C.G of the area & S is the area.

P = Resultant pressure $= \sum g\rho x \delta S = g\rho \Sigma x \delta S$

& $\bar{x} = \dfrac{\Sigma x \delta S}{\Sigma \delta S} = \dfrac{\Sigma x \delta S}{S}$.

i.e. $P = g\rho \bar{x} S$.

(2) Show that the C.P of a plane area ~~always~~ is below the centroid & approaches it as the surface is immersed more deeply in the liquid

Taking axes as shown in the figure

$x = y \sin\theta$ & $p = g\rho y \sin\theta$.

$\bar{x} = \dfrac{\Sigma xy\delta S}{\Sigma y \delta S}$; $\bar{y} = \dfrac{\Sigma y^2 \delta S}{\Sigma y \delta S}$

If h = y-coord of C.G, $h = \frac{\Sigma y \delta s}{\Sigma \delta s}$ i.e. $hA = \Sigma y \delta s$

$\therefore \quad \bar{y} = \frac{\Sigma y^2 \delta s}{Ah} = \frac{Ak_1^2}{Ah} = \frac{k_1^2}{h}$ [Ak_1^2 = M. I about x-axis

If k = rad. of gyration about a ll^{el} to ox through centroid of area

$$k_1^2 = k^2 + h^2$$

$\therefore \quad \bar{y} = \frac{k^2}{h} + h.$ i.e. $\frac{k^2}{h}$ = distance of C.P from C.G measured ll^{el} to OY

k^2/h is +ve $\therefore$ C.P is below C.G.

k^2 is const & as h increases, $\bar{y} \to h$. i.e C.P $\to$ C.G.

p.15. By $\vec{q}$ the velocity of the fluid at the section is meant the mean velocity for the different points of the cross-section which is small.

p.21. Exc. 1. – Velocity at the lower level is given by noting that diameter at lower level is equal to $\left(d + \frac{2}{5}d\right)$ where d = diameter at upper level. Hence vel. at lower level = vel. at upper level $\times \frac{5}{7} = \frac{28}{5} \times \frac{5}{7} = 4$ ft/sec.

From Bernoulli's eqn

$$\frac{(5.6)^2}{2g} + 0 + \frac{20.5}{g\rho} = \frac{(4)^2}{2g} + 4 + \frac{p}{g\rho}$$

& this gives p at lower level.

Ex. 2. $h_p = \frac{\gamma}{\gamma - 1} \frac{p_0}{g\rho_0} \left\{ \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}$, omitting $\left[-\frac{\gamma}{\gamma-1} \frac{p_0}{g\rho_0} \right]$ which is a const.

we have $\frac{p_0}{g\rho_0} \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{p}{g\rho_0} \left(\frac{p_0}{p}\right)^{\frac{1}{\gamma}} = \frac{p}{g\rho_0} \cdot \frac{\rho_0}{\rho} = \frac{p}{g\rho}$.

Hence $\frac{q^2}{2g} + h_p$ = const gives $q^2 + \frac{2\gamma}{\gamma-1} \frac{p}{\rho}$ = const.

In (27), check carefully that $h_p = h_{p_0}$ for $p = p_0$.

p. 20 – 2 lines above eqn (31).

Here $\frac{q_0^2}{2g} + h_{p_0} = h_{p_s}$ $\left[\frac{q_0^2}{2g} + h_{p_0} = h_{p_s}\right]$ ——— (A)

$$\frac{q_0^2}{2g} = \frac{r q}{r-1} \cdot \frac{p_0}{g \rho_0}\left\{\left(\frac{p_s}{p_0}\right)^{\frac{r-1}{r}} - 1\right\}$$

Since h_p is chosen such that $h_s = 0$ for $p \neq p_0$.

$$\frac{q_0^2}{2} + \frac{rq}{r-1} \cdot \frac{p_0}{\rho_0} = \left(\frac{p_s}{p_0}\right)^{\frac{r-1}{r}}$$

$$\left(\frac{r-1}{g_0 r}\right)\frac{p_0}{\rho_0}\left(\frac{\rho_0 q_0^2}{2 p_0}\cdot\frac{r-1}{r_1} + 1\right) = \left(\frac{p_s}{p_0}\right)^{\frac{r-1}{r}}$$

$$\frac{p_s}{p_0} = \left(1 + \frac{\rho_0 q_0^2}{2 p_0}\cdot\frac{r-1}{r}\right)^{\frac{r}{r-1}}\left(\frac{r-1}{r}\cdot\frac{p_0}{\rho_0}\right)^{\frac{r}{r-1}}$$

we have
Then (27) becomes $h_{p_0} = \frac{r}{r-1}\frac{p_c^{\frac{1}{r}}}{g\rho_0} p_0^{\frac{r-1}{r}} + \text{const}$

$$\text{i.e. const} = h_{p_0} - \frac{r}{r-1}\cdot\frac{p_0}{g\rho_0}$$

$$\therefore (27) \text{ becomes } h_{p_s} = \frac{r}{r-1}\left\{\frac{p_0^{\frac{1}{r}}}{g\rho_0} p_s^{\frac{r-1}{r}} - \frac{p_0}{g\rho_0}\right\} + h_{p_0}$$

From eqn (A) $\frac{q_0^2}{2} + h_{p_0} = h_{p_0} + \frac{r}{r-1}\{\quad\}$

$$\frac{\rho_0 q_0^2}{2 p_0} = \frac{r}{r-1}\left\{\frac{p_0}{g\rho_0}\left(\frac{p_s}{p_0}\right)^{\frac{r-1}{r}} - \frac{p_0}{g\rho_0}\right\}.$$

$$\left\{\frac{\rho_0 q_0^2}{2 p_0}\cdot\frac{r-1}{r} + 1\right\} = \left(\frac{p_s}{p_0}\right)^{\frac{r-1}{r}}$$

$$\left\{\quad\right\}^{\frac{r}{r-1}} \text{ i } \frac{p_s}{p_0}$$

$$\frac{p_s}{p_0} = 1 + \frac{rq}{r-1}\cdot\frac{\rho_0 q_0^2}{2 p_0}\cdot\frac{r-1}{r} + \frac{\frac{r}{r-1}\left(\frac{r}{r-1}-1\right)}{1\cdot 2}\cdot\frac{(\rho_0 q_0^2)^2}{4 p_0^2}\left(\frac{r-1}{r}\right)^2 + \cdots$$

a result by Binomial theorem

$$= p_0 \left\{ 1 + \frac{\rho_0 g_0}{2 p_0} + \frac{1}{8\gamma} \frac{(\rho_0 g_0^2)^2}{p_0^2} + \cdots \right\} \simeq p_0 + \tfrac{1}{2} \rho_0 g_0^2 + \frac{1}{8\gamma} \frac{\rho_0^2 g_0^4}{p_0} + \cdots$$

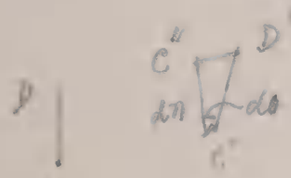
which is (31).

p.22. In derivation of (32) & (33), note that thickness 1^{st} to plane is unity ie $@$ $ds\,dn\,1$ is volume element. ~~We integrate $\frac{dH}{dn}$ from $H = \frac{q^2}{2g} + h + h_p$~~

~~but here $\frac{dH}{dn} = \frac{K}{g} \frac{dq}{dn}$~~

p.23

If $d\theta$ = angle between CD & C'D', the distance ahead is $= dn \cdot \tan d\theta$
$= dn \cdot d\theta$



$$\therefore d\theta = \frac{\text{dist. ahead}}{dn} = \frac{dq}{dn} \cdot dt.$$

$$\therefore \frac{d\theta}{dt} = dq/dn$$

Angular vel. q/R of segment AB is Counterclockwise $(+ve)$ $= \frac{q}{R}$.

" dq/dn " CD Clockwise $(-ve)$ $= -\frac{dq}{dn}$.

Average angular vel = rotation of element @

$q/R =$

Lectures on Fluid Mechanics

to 6th T.S.O'3

(1) 17/1/57 – Introduction as in 1st lecture to 5th T.S.O's as on p. 17 – 4 steps in physical theory – perfect fluids (+ viscosity) – laminar & turbulent motion – stress (normal & shear – The latter zero for fluid in eqm), pressure – normal stress same for every surface element at P – Eqm of liquid under gravity ($p_0 = p_1 + g\rho h$)

(2) 19/1/57 – Eqm of liquid under gravity ($p_2 = p_1 + g\rho h$) – heterogeneous liquid $\rho_1 = \rho_2$ in hor. layers – Eqm of gas under gravity: atmosphere

(3) 21/1/57 – The atmosphere – polytropic case – graphical method – examples

(4) 22/1/57 – Eqm of fluid in arbitrary field – Equi-pressure & equipotential surfaces – Rotating vessel containing fluid – Resultant pressure & C.P – applns of calculus to ~~[illegible]~~ & C.P.

(5) 23/1/57 – Pressure on curved surface – metacentre

(6) 24/1/57 – metacentre – Examples (p. 11) – [Exs. 5·34, p. 76; Exc. 6, p. 88] of Ramsey.

(7) 25/1/57 – further examples of Hydrostatics.

(8)–(10) – missed as for Patnaiks time-table.

Eq. of fluid motion with viscosity

(11) – 28/1/57 – Lagrangian & Eulerian methods – path lines & stream lines – steady motion – continuity – stream filaments – Bernoulli's eqn – with compressibility – Torricelli's theorem – simple pitot-Pitot tube.

(12) – 29/1/57 – above & space – Examples p. 21

(13) – 30/1/57 – further examples p. 21.

(14) – 31/1/57 – lecture given to Prabhakar

(15) – 1/2/57 – formula for $\frac{dH}{dn}$ – Rotation of element

(16) – 2/2/57 – Period used for Mathematics

(17) – 4/2/57 – Circulation & rotation – Kelvin's theorem

(18) – 5/2/57 – Vel. potential & stream function – source & sink

P.24 · Eq. (11) $\frac{(5.6)^2}{2g} + 0 + 20.5 = \frac{q^2}{2g} + 4 + p$ whence

p is found.

Light
q at right main
$= \frac{5}{1} \cdot (5.6)$
$= 4$

(2) $\frac{q^2}{2g} + h_p = \text{Const}, \quad \frac{q^2}{2g} + \frac{\gamma}{\gamma-1} \cdot \frac{p_0}{g\rho_0}\left\{\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} = \text{const}$

$$\frac{q^2}{2g} + \frac{\gamma}{\gamma-1} \frac{p_0}{g\rho_0}\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \text{const}$$

$$q^2 + \frac{2\gamma}{\gamma-1} \cdot \frac{p_0}{\rho_0}\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \text{const}$$

Substituting for ρ_0 from $\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma}$ we get required result

$\gamma = 1.5$.

(3) $q^2 + \frac{2c^2}{\gamma-1} = \text{const}.$

Proof: ~~Proof~~ ~~As the velocity of sound~~

$$q^2 + \frac{2}{\gamma-1} c^2 = q_0^2 + \frac{2}{\gamma-1} c_0^2$$

$$\rho q \Gamma = \rho_0 q_0 \Gamma_0.$$

by log. diffn: $\frac{1}{\Gamma}\frac{d\Gamma}{ds} = -\frac{1}{q}\frac{dq}{ds} - \frac{1}{\rho}\cdot\frac{d\rho}{dp}\cdot\frac{dp}{ds}$

$$= -\frac{1}{q}\frac{dq}{ds}\left(1 - \frac{q^2}{c^2}\right)$$

$\frac{dp}{d\rho} = \frac{\gamma p}{\rho}$

$\uparrow$

$c = \left(\frac{\gamma p}{\rho}\right)^{\frac{1}{2}} = \left(\frac{dp}{d\rho}\right)^{\frac{1}{2}}.$

$\frac{1}{\rho}\cdot\frac{dp}{ds} = q.$

$\left[\frac{d}{ds}\left(\frac{q^2}{2g}\right) + \frac{1}{g\rho}\frac{dp}{ds}\right.$

$q\frac{dq}{ds} = -\frac{1}{\rho}\frac{dp}{ds}$

Gauss: $\iiint_V \operatorname{div}\vec{A}\cdot dV = \iint_S A_n\, dS$

Stokes: $\oint_C \vec{A}\cdot d\vec{r} = \iint_S (\vec{n}\cdot\operatorname{curl}\vec{A})\, dS$

Green: $\iiint_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\, dV = \iint_S \left(\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right) dS$

(19) – 6/2/57 – Doublet – Vortex – Cylinder in uniform stream

(20) – 7/2/57 – Lecture not taken

(21) – 8/2/57 – Vortex – Cylinder in uniform stream.

(22) – 9/2/57 – cylinder in uniform stream with circulation – other examples (1) – images – simple source w.r.t. a plane & a sphere – (2) a doublet w.r.t. a sphere – $w = \frac{m}{2\pi i} \log \frac{z^2 - a^2}{z^2 + a^2}$ – $z = c \cosh w$ – also vortex with circulation – lift.

(23) – 11/2/57 – above two examples – Kutta – Joukowski theorem.

(24) – 12/2/57 – $z = c \cosh w$ – Complex variable – Cauchy's theorem – theorem of residues – Blasius' theorem – Kutta–Joukowski theorem.

(25) – 15/2/57 – Elem. Vector Analysis.

(26) – 16/2/57 – Did mathematics

(27) – 18/2/57 – Some more vector analysis – Stokes', Gauss' & Green's theorem – Examples of vectors in hydrodynamics – vorticity = $\mathrm{rot}\, \vec{v}$ – circulation = $\int_C \vec{v} \cdot d\vec{r}$; deduction from Stokes' theorem that $\vec{v} = -\mathrm{grad}\, \phi$ – eqn of continuity $\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \vec{v}) = 0$; $\rho = \text{const}$, $\mathrm{div}\, \vec{v} = 0$. ($\nabla^2 \phi = 0$) – Examples: $\mathrm{div}(\phi \vec{A}) = \phi\, \mathrm{div}\, \vec{A} + \vec{A} \cdot \mathrm{grad}\, \phi$,

$\mathrm{rot}(\phi \vec{A}) = \phi\, \mathrm{rot}\, \vec{A} - \vec{A} \times \mathrm{grad}\, \phi$; $\mathrm{div}(\vec{A} \times \vec{B}) = \vec{B}\, \mathrm{rot}\, \vec{A} - \vec{A}\, \mathrm{rot}\, \vec{B}$; $\mathrm{rot}(\vec{A} \times \vec{B}) = (B \mathrm{grad}) A - (A \mathrm{grad}) B - B\, \mathrm{div}\, A + A\, \mathrm{div}\, B$

$\mathrm{grad}(\vec{A} \cdot \vec{B}) = \vec{A} \times \mathrm{rot}\, \vec{B} + (\vec{A} \mathrm{grad}) \vec{B} + \vec{B} \times \mathrm{rot}\, \vec{A} + (\vec{B} \mathrm{grad}) \vec{A}$; $\mathrm{grad}(\phi \psi) = \phi\, \mathrm{grad}\, \psi + \psi\, \mathrm{grad}\, \phi$

(28) – 19/2/57 – General remarks about fluid mechanics, friction, inertia & pressure, Ozeen & Prandtl, boundary layer – Examples further examples of fluid motion –

(i) $w = Vz$, (ii) $w = Vz^n/n$ ($n = 3$, $K = 0$, 1 walls & $K = 0$, 2 walls – $n = 2$, $K = 0$, 1 walls – $n = 3/2$, $K = 0$, 1 walls, $K = 1$, 2 walls – $n = \frac{2}{3}$, $K = 0$, 1 walls – $n = \frac{1}{2}$, $K = 1$ walls – $|\vec{w}| = a r^{n-1}$ at $z = 0$, $w = 0$, $n > 1$, $w = \infty$ inside $n = 1$, $w = \infty$, $n < 1$, in this case viscosity acts) (vii) – $n = -1$, doublet – doublet + uniform flow, $w = V(z + \frac{1}{z})$, $\psi = \text{const}$, $V \sin\theta (r - \frac{1}{r}) = \text{const}$, $\psi = 0$ gives $\theta = 0$ or π or $r - \frac{1}{r} = \infty$ i.e. $r = 1$ i.e. circle – (iv) $w = V \log z$ ($n \to 0$), $w = iV \log z$ (vortex), (v) $w = Ve^{\alpha z}$, $\psi = 0 \to \alpha y = K\pi$ ($K = 0, 1, 2 \ldots$) $\psi = \text{const}$ hyp. $K = n$ & $n+1$, $e^{-\alpha x} = V \sin \alpha y / \text{const}$ or $x = -\frac{1}{\alpha} \log(\sin \alpha y) + \text{const}$. $w = Ve^{i\alpha z}$ as following uniform motion of small amplitude. (vi) Conformal transformations: $z = t + 1/t$, $z = i(t + \frac{1}{t})$ (plate), $z = a(t + \frac{1}{t}) + ib(t + \frac{1}{t})$ ($\alpha = \tan^{-1}(-b/a)$ – Joukowski profile etc..

$t = z + \frac{1}{z}$.

$|z| = 1$ $\quad t = re^{i\theta} + \frac{1}{r}e^{-i\theta}$

$t = e^{i\theta} + e^{-i\theta} = 2\cos\theta$.

$\xi + i\eta = 2\cos\theta$.

$\eta = 0, \ \xi = 2\cos\theta$.

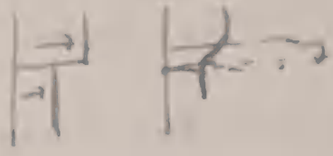
i.e. ξ varies from -2 to $+2$

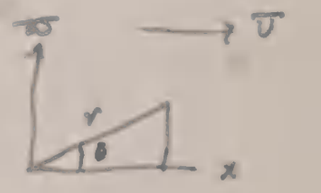
$w = Vt \quad f(t) = Vt = w$.

$w = V\left(z + \frac{1}{z}\right)$.

$\log z$

$\log re^{i\theta}$





Sphere in uniform stream: Ox taken as symmetry axis.

(i) Uniform stream – $\phi = -Ux$, $\psi = -\frac{1}{2}Ur^2\sin^2\theta$

(ii) Source at O .. $\phi = \mu/r$, $\psi = \mu x/r$

(iii) Doublet (along Ox) .. $\phi = \mu x/r^3$, $\psi = -\mu\sin^2\theta/r^3$.

(iv) Sphere in stream .. $\psi = \frac{1}{2}Ur^2\sin^2\theta\left(1 - a^3/r^3\right) \quad (a^3 = 2\mu/U)$

[For cylinder: l/r = shape]

Inverse flow: For given shape & orientation, $F \propto f(\mu, \rho, V, l)$. ($l$ denotes size).

Let $MLT^{-2} = \mu^a \rho^b V^c l^d$. Putting $F = \frac{1}{2}\rho V^2 l^2 f\left(Vl/\nu\right) = \frac{1}{2}\rho V^2 l^2 \cdot C_F = \frac{1}{2}\rho V^2 A \cdot C_F$

$F = q \cdot A \cdot C_F$

$\frac{Vl}{\nu} = \frac{\rho Vl}{\mu} = \frac{[ML^{-3}LT^{-1}L]}{[ML^{-1}T^{-1}]} = \frac{[ML^{-1}T^{-1}]}{[ML^{-1}T^{-1}]} =$ pure number $= R$.

$C_F = f(R)$:

$v = f(\mu, \rho, V, l)$ fails $\quad v = V\phi(R)$. $\quad v/V = \phi(R)$

Navier-Stokes eqn: $\rho \frac{D\vec{q}}{Dt} = \rho\vec{F} - \text{grad}\, p + \frac{1}{3}\mu\,\text{grad}\,\text{div}\,\vec{q} + \mu\nabla^2\vec{q}$ //

For incompressible fluids: $\text{div}\,\vec{q} = 0$. $\quad \frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho}\text{grad}\, p + \vartheta\nabla^2\vec{q}$ //

[$\frac{D\rho}{dt} + \rho\,\text{div}\,\vec{q} = 0$ Continuity eqn]

[$\frac{\partial\rho}{\partial t} + \text{div}(\rho\vec{F}) = 0$ or]

(29) – 20/2/57 – Previous lecture portions contd. (50)

(30) – 21/2/57 – Class missed.

(31) – 22/2/57 – Vortex motion – $\omega = \frac{1}{2}\left(\frac{q}{R} - \frac{dq}{dn}\right) = \frac{1}{2}\operatorname{rot}\vec{q}$ – $\Gamma = \int_C \vec{q}\cdot d\vec{r} = \int_S \operatorname{rot}\vec{q}\cdot dS$ – for irrotational motion $\operatorname{rot}\vec{q}=0$ & $\Gamma=0$ for closed circuit – In vortex motion $\operatorname{rot}\vec{q}\neq 0$, $\operatorname{rot}\vec{q} = 2\vec{\omega}$, $\Gamma\neq 0$, $\operatorname{div}\vec{\omega}=0$ – $\int \vec{q}\cdot d\vec{r} = \text{const}$ – Vortex lines & Vortex filaments – Helmholtz's Theorems (Thomson) (non-viscous fluids) – Relation between vortex motion & surface of discontinuity – boundary layer sticking or not sticking – Ex. of sharp trailing edge. ($\operatorname{rot}\vec{q} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$) – surface of discontinuity as a vortex surface ($\mu\to 0$, layer → surface) – Explanation of formation of circulation round an aerofoil

Momentum theorem – Applications of the same – mention of Kutta–Joukowski theorem.

(32) – 25/2/57 – Mention of A.M. Theorem for control surface – Consequences of K-J-Theorem – ~~drilling~~

(33). For cases when $q_1 = \infty$ at edge, mention $W = f(z) = \frac{a}{n} z^n$, $n = 2/3$ & $n = \frac{1}{2}$.

(34) – 25/2/57 – Viscosity – p_{hx}, p_{hy}, p_{hz} – tensor(p) – τ (viscous stress) $= \mu \frac{du}{dy}$; $\mu = ML^{-1}T^{-1}$, [Hagen–Poiseuille law] $\nu = \mu/\rho = L^2T^{-1}$ – mention of Navier–Stokes Partial diff eq^{n} & Reynolds's number.

To obtain F single series with varying V will do for ie keeping μ, ρ, l const will also mean keeping $\underline{A}$ const ($\because l/\sqrt{A}$ fixes shape) & also orientation const & $F = \frac{1}{2}\rho V^2 A C_F$ & hence A & C_F will be const – $F/\frac{1}{2}\rho A$ depends on shape, orientation & R – ~~Knowing~~ stream pattern depends only on R [from $\nabla = \nabla\phi(R)$] – Generalisation from V to boundary cond^{n} – similar flows.

R = 0 for ideal fluid

(35) – 26/2/57 – Portions left over from previous lecture – Laminar & turbulent flow – hydraulic hypothesis.

(36) – 28/2/57 – hydraulic hypothesis & Navier Stokes eq^{n} – boundary layer + N-S. eq^{ns} – hydraulic hypothesis & discontinuous perfect fluid motions, separation points, tendency towards separation even before –ve pressures are reached; 250° & 120° in case of cylinder for A smaller & larger R – accg. to hyd. hypothesis each type of discontinuous perfect fluid motion is possible in a real fluid flow (vortex sheet idea also made here); which one actually occurs ~~often~~ to depends on R ie on μ & hence the boundary layer theory appears to play a part – Boundary layer theory (1) As R ↑ increases motion in free stream is turbulent & average flow → perfect

Hagen - Poiseuille law: N-Stokes: $\frac{D\vec{v}}{Dt} = \vec{F} - \frac{1}{\rho}\operatorname{grad} p + \nu \nabla^2 \vec{v} + \frac{1}{3}\nu \operatorname{grad}\operatorname{div}\vec{v} + \nu \nabla^2 \vec{v}$.

C_L, S & C_M determine by profile & R & α_0.

Centre of pressure: $\begin{cases} C_L = k\alpha & k \text{ on } R \\ C_D = a + \frac{C_L^2}{b} & a \text{ on shape, } b \text{ on } R \text{ -- profile a induced drags.} \\ C_M = \text{const for aerodynamic centre (focus). (shape const value on shape of (wh to d = const).} & \text{(centre of pressure)} \end{cases}$

Polar diagram:

$$V = -\frac{\Gamma}{4\pi r}$$

$$dV = -\frac{\Gamma}{4\pi r^2} dl = \frac{\Gamma}{4\pi r^2} \cdot dS \cdot \sin\alpha = \frac{\Gamma\, dS}{4\pi} \cdot \frac{R}{r^3}.$$

$\frac{dS}{dl} \triangle dl$

$p = f(\rho)$

$c = \sqrt{E/\rho}$ by defn $\qquad E = \frac{dp}{-dV/V} = \frac{\rho \cdot dp}{d\rho}$ $\qquad \rho V = M$, $\frac{d\rho}{\rho} = -\frac{dV}{V}$

$E/\rho = \frac{dp}{d\rho}$ i.e. $c^2 = \frac{dp}{d\rho}$

Euler's eqn $\rho \frac{d\vec{v}}{dt} = -\operatorname{grad} p$ or $\rho \frac{d\vec{v}}{dt} = -c^2 \operatorname{grad}\rho$.

Continuity eqn: $\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{v}) = 0$.

Bernoulli's eqn: $\frac{1}{2}|\vec{v}|^2 + P(\rho) = \text{const}$ along path of particle:

To every $|\vec{v}|$ is assigned a ρ & hence a c. If $v = c$, $v = c^*$, if $|\vec{v}| < c^*$ subsonic $|\vec{v}| > c^*$ supersonic

fluid motion a boundary layer becomes necessary. Inside boundary layer also motion be turbulent; in case of plate $\delta_{@} = 3.65\sqrt{\nu x/V}$ & at trailing edge $\delta = \max = \delta_t = 3.65 l/\sqrt{R}$, true upto $R = 250,000$ to $300,000$ — above this motion in boundary layer becom upto this calculations made on laminar steady solns N-Stg^{n}s — above this motion in boundary layer becomes turbulent. using laws of turbulent motion Karman fave modifications valid upto $R = 300000$ to 400000 — boundary layer theory & separation points —

Drag given by $D_p = \frac{1}{2}\rho V^2 A \cdot C_p$; C_p = coeffn of drag depends on shape & R; $C_p = \frac{D_p}{\frac{1}{2}\rho A}$

(37) – 1/3/57 – Aerofoils – Geometry – Three main results – Theoretically developed aerofoils.

(38) – 2/3/57 – Joukowski trn $z = z_1 + b^2/z_1$ – Karman-Trefftz profiles – C_L, C_D, C_M – recapitulation of results for infinite wing span

(39) – (40) – 4/3/57 – Von Mises profiles – meta centric parabola – vortex motion, rectilinear vortices, $w = i\frac{\Gamma}{2\pi}\log z$, vortex lines, vortex sheet & surface of discontinuity, Ex. of plate vortices leaving at two ends, (cumulative Γ) – curved vortex lines, $\Gamma_1 d\sigma_1 = \Gamma_2 d\sigma_2$, vortex lines closed or $\to\infty$ – $dV = \frac{\Gamma}{4\pi}\frac{R}{r^3}ds$ – Electrical analogy [vortex filament → electric circuit, $\Gamma \Rightarrow$ Elec. C, $V \to$ magnetic force, vortex & vortex to magnetic poles] – horse shoe vortex & induced drag – bundle of lifting lines & associated trailing vortices – Elliptic lift distn – $C_L^2/C_D = \pi AR$ – wings of elliptic plan form – $C_L \sim 6\alpha'/1+2/AR$ – $C_D = C_{D_p} + C_{D_i}$ [$C_{D_i} = 0$ for infinite span] ie $C_D = C_{D_p} + C_L^2/\pi AR$] – Theorem on p. 253 of von Mises – Stalling point is the upper end of the straight part of the C_L–α' curve – stalling & wake – $V \propto \frac{1}{\sqrt{C_L}}$. – (increase of profile drag) – Condⁿ for max C_L max; $C_{L\max}$ has plt, $\propto$ Camber $\propto R$.

High lift devices @

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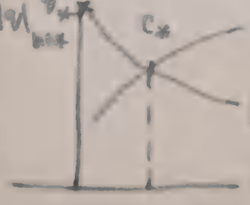
(41) – 5/3/57 – Portions left over – compressibility, $c^2 = dp/d\rho$ – subsonic & supersonic velocities – Mach cone

(A) $= de + p\,d\tau$. ($\tau = \rho^{-1}$).

(42) – 6/3/57 – $T\cdot ds = dh + p\,dV = dh + p\,d\rho^{-1}$: for $ds = 0$, $dh + p\,d\rho^{-1}$; $P = h + pV$ = heat content $= e + p\tau$ per unit mass: $dP = dh + p\,d\rho^{-1} + \rho^{-1}dp$ for $ds = 0$ – $p = \sigma\rho^{\gamma}$ [$\sigma = \sigma(s)$, $\gamma = 1.4$ for air] giving (with dfty) $P = c^2/\gamma - 1$ – $c \& c_*$ – $dP = dp/\rho$ ie if $dp > 0$, $dP > 0$ ie if pressure increases with density, P increases with ρ



for plot of $|q|$ against $P(\rho)$ & c increases with P [from $c^2 = (\gamma-1)P$] – $P(0) = 0$

If $|q| < c_*$, $|q| < c$ – right
If $|q| > c_*$, $|q| > c$ – left

$\Big\}$ $-\frac{1}{2}|q|^2 + P(\rho) = \frac{1}{2}\rho q_*^2 = P_*$ – Bernoulli's eq^{n}

C_* defined by $u = c = c_*$ & from Bernoulli eqn $\frac{1}{2}c_*^2 + \frac{1}{\gamma-1}c_*^2 = \frac{1}{2}q_*^2$ or $c_*^2 = \frac{\gamma-1}{\gamma+1}q_*^2$ $(c_* < q_*)$ (58)

(B) Nozzle Flow:

$$\left.\begin{aligned} \rho u A &= Q = \text{const} \\ \frac{1}{2}u^2 + p &= \frac{1}{2}q_*^2 \end{aligned}\right\}$$

$$\left(\frac{u^2}{c^2} - 1\right)\frac{du}{u} = \frac{dA}{A}.$$

$A(x) = \min$ for $x = x_*$
$A = \infty$ at $x = x_0$.

(i) c^* not reached before $x = x_*$

(ii) u increases as long as A decreases unless $u = c_*$ is reached

For $u = 0$, $-\frac{du}{u} = \frac{dA}{A}$ near x_0 (unless $u = c = c_*$ for $x < x_*$)

(iii) Two cases: A: c_* not reached at $x = x_*$ (at $x = x_*$)

$\therefore dA = 0 \rightarrow du = 0$ i.e. $u = u_{max} < c_*$

for $x > x_*$, $dA > 0$. $\therefore du < 0 \therefore u^2/c^2 - 1 < 0$.

i.e. u decreases within subsonic flow

B: (a) c_* reached at $x = x_*$ & $du < 0$ for $x > x_*$ ($u = u_{max} = c^*$ at x_* increasing to decrease)

for $x > x_*$, $dA > 0$, $du < 0$ mean $u < c_*$ i.e. $u < c$ i.e. $u^2/c^2 - 1 < 0$.

subsonic flow exists at x_*

(b) c_* reached at $x = x_*$ & $du > 0$ for $x > x_*$.

for $x > x_*$, $dA > 0$; $u^2/c^2 - 1 > 0$, u increases monotonically - changes from subsonic to supersonic at x_*.

Verifying B(b) from $\frac{d^2p}{du^2} = \rho(M^2 - 1)$. - not perfectly rigorous since variation of mean ρu & not of u is considered.

Special case of $p = C\rho^\gamma$:

(43)-(44) — 7/3/57 — recapitulation — relation $A = Q\,f(p/p_0)$ in case of $p = C\rho^\gamma$ — S-p curve of different values of Q (i) $Q > Q_*$, (ii) $Q = Q_*$ (2 values of p_e), (iii) $p_e < p''$ — free jet within (iv) $p'' < p_e < p'$ ($p_e > p'$, p'')

"Compression shock" — one dimensional shocks in piston moving fast

(45) — 8/3/57 — To get (111) — from (106), $p_0 - p_1 = \rho_1 v_1^2 - \rho_0 v_0^2 = m(v_1 - v_0)$ using (105)

To get (112) — from (105) $\rho_0 v_0 v_1 = m v_1$ & $\rho_1 v_0 v_1 = m v_0$. Subtracting & using (111) we get (112)

To get (113) vide track of p. 58 of V.R.T.

To get (114): $dP = dp/\rho$ when $d\rho = 0$, $p = \sigma\rho^\gamma$, $dp = \sigma\gamma\rho^{\gamma-1}d\rho$

i.e. $dP = \sigma\gamma\rho^{\gamma-2}d\rho$ i.e. $P = \frac{\sigma\gamma}{\gamma-1}\cdot\rho^{\gamma-1} = \frac{\gamma}{\gamma-1}\cdot\frac{p}{\rho}$.

$$\frac{\gamma p}{\rho} = c^2 = \frac{dp}{d\rho} = \sigma\gamma\rho^{\gamma-1}; \quad P = \frac{\sigma\gamma}{\gamma-1}\rho^{\gamma-1}$$

i.e. $\frac{P}{c^2} = \frac{1}{\gamma-1}$ or $P = \frac{1}{\gamma-1}c^2$.

$\frac{1-\mu^2}{2\mu^2} = \frac{1-\frac{(\gamma-1)^2}{(\gamma+1)^2}}{2\frac{(\gamma-1)^2}{(\gamma+1)^2}} = \frac{(\gamma+1)^2-(\gamma-1)^2}{2(\gamma-1)^2}$

& $\frac{1-\mu^2}{2\mu^2} = \frac{1-(\gamma-1)/(\gamma+1)}{2(\gamma-1)/(\gamma+1)} = \frac{(\gamma+1)-(\gamma-1)}{2(\gamma-1)} = \frac{1}{\gamma-1}$.

To get (116): from (110) using (114)

$$\frac{1}{2}v_0^2 + \frac{1-\mu^2}{2\mu^2}c_0^2 = \frac{1}{2}v_1^2 + \frac{(1-\mu^2)}{2\mu^2}c_1^2 = \frac{1}{2}q_{max}^2$$

multiplying by $2\mu^2$: $\mu^2 v_0^2 + (1-\mu^2)c_0^2 = \mu^2 v_1^2 + (1-\mu^2)c_1^2 = c_*^2$

To get (117)'. from Bernoulli's eqn: $\frac{1}{2}v^2 + \frac{p}{\rho} = \text{const}$

i.e. $\frac{1}{2}q^2 + \frac{p}{\rho} = \frac{1}{2}q_{max}^2 = \mu^2 c_*^2 \cdot \frac{1}{2}$

i.e. $q^2 + 2\cdot\frac{p}{\rho} = \mu^2 c_*^2$ and $c^2 = \frac{dp}{d\rho} = \sigma\gamma\rho^{\gamma-1} = \frac{\sigma\gamma\rho^\gamma}{\rho} = \frac{\gamma p}{\rho}$.

$q^2 + \frac{2c^2}{\gamma} = \mu^2 c_*^2$ or $q^2 + \frac{2(1-\mu^2)}{1+\mu^2}c^2 = \mu^2 c_*^2$.

$\frac{\gamma-1}{\gamma+1} = \mu^2$

$\gamma = \frac{1-\mu^2}{1+\mu^2}$

$\gamma = \frac{1+\mu^2}{1-\mu^2}$

$\therefore q^2 - c_*^2 = 1$ $\quad q^2(1+\mu^2) + 2(1-\mu^2)c^2 = \mu^2 c_*^2$

$\mu^2(q^2 - c_*^2) = 2(\mu^2-1)q^2 - q^2$.

Bernoulli's eqn is $\frac{1}{2}q^2 + P = \frac{1}{2}q_*^2$ & $P = \frac{1}{\gamma-1}c^2 = \frac{1-\mu^2}{2\mu^2}\cdot c^2$.

$\therefore q^2 + \frac{(1-\mu^2)}{\mu^2}c^2 = c_*^2/\mu^2$

$q^2\mu^2 + (1-\mu^2)c^2 = c_*^2$

$\therefore q^2 - c_*^2 = q^2 - q^2\mu^2 - (1-\mu^2)c^2$

$= (1-\mu^2)(q^2 - c^2)$

To get (118): from (112) $v_1 v_0 = \dfrac{p_1 - p_0}{\rho_1 - \rho_0}$ ~~$= \sqrt{\dfrac{p_0 v_0^2 + p_1 v_1^2}{\rho_1 - \rho_0}}$~~ using (106).

~~From~~ $M^2 = v^2/c^2$ from (116), $\mu^2 v_1^2 + (1-\mu^2)\dfrac{p_1}{\rho_1}\gamma = c_*^2$

$\mu^2 v_0^2 + (1-\mu^2)\dfrac{p_0}{\rho_0}\gamma = c_*^2$ } using $c^2 = \dfrac{\gamma p}{\rho}$

ie $\mu^2(\rho_0 v_0^2 - \rho_1 v_1^2) + \gamma(1-\mu^2)(p_0 - p_1) = (\rho_0 - \rho_1)c_*^2$

& using (106): $\mu^2(p_0 - p_0) + \gamma(1-\mu^2)(p_0 - p_1) = (\rho_0 - \rho_1)c_*^2$

ie $\dfrac{p_1 - p_0}{\rho_1 - \rho_0}\left\{-\mu^2 + \gamma(1-\mu^2)\right\} = c_*^2$ & $-\mu^2 + \gamma(1-\mu^2) = 1$

ie $v_0 v_1 = c_*^2$.

To get (118)': $(\tau_0 - \tau_1)\dfrac{p_0 + p_1}{2} = \rho_0 e_1 - e_0$

$p = \dfrac{\gamma}{\gamma - 1} p\tau = \dfrac{1+\mu^2}{2\mu^2} p\tau$

$\rho = \bar{\rho} + \rho'\tau \qquad e = \bar{p} - p\tau$

$\therefore e_1 - e_0 = (\bar{p}_1 - \bar{p}_0) - (p_1\tau_1 - p_0\tau_0)$

$= \dfrac{1+\mu^2}{2\mu^2}(p_1\tau_1 - p_0\tau_0) - (p_1\tau_1 - p_0\tau_0)$

$\therefore (\tau_0 - \tau_1)\left(\dfrac{p_1 + p_0}{2}\right) = (p_1\tau_1 - p_0\tau_0)\left(\dfrac{1-\mu^2}{2\mu^2}\right) = (p_1\tau_1 - p_0\tau_0)\left(\dfrac{1-\mu^2}{2\mu^2}\right)$

ie $\mu^2(\tau_0 - \tau_1)(p_0 + p_1) = (1-\mu^2)(p_1\tau_1 - p_0\tau_0)$

which leads to (118').

(119): is an immediate consequence of (118').

(120): " (119)

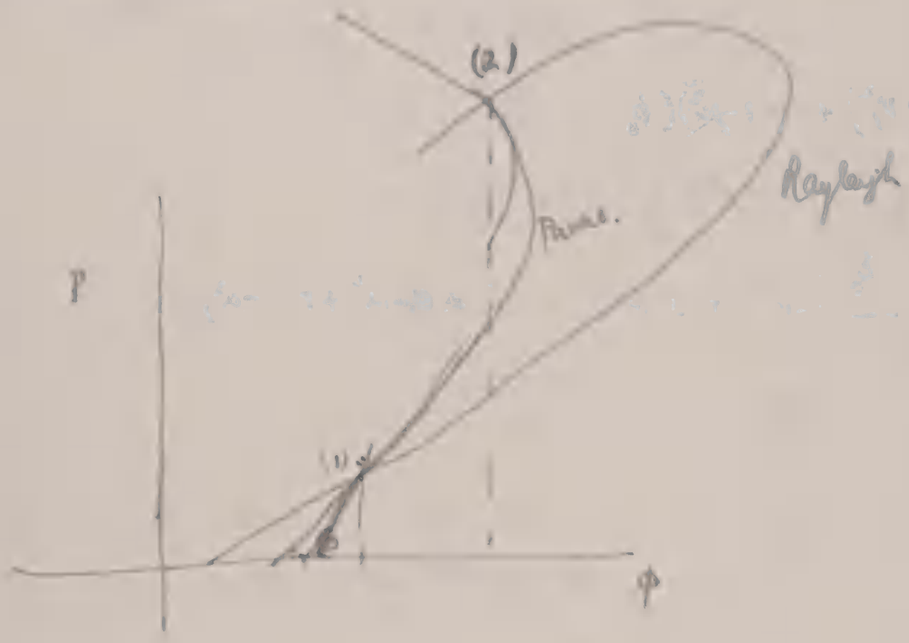
(121): $\dfrac{p_1 - p_0}{p_0} = \dfrac{(1+\mu^2)(\rho_1 - \rho_0)}{\rho_0 - \mu^2\rho_1}$ from (119)

using $\rho_0 v_0 = \rho_1 v_1$ ie $\rho_1 = \dfrac{\rho_0 v_0}{v_1}$ this $= \dfrac{(1+\mu^2)\left(\dfrac{\rho_0 v_0}{A} - \rho_0\right)}{\rho_0 - \mu^2\rho_0 v_0/v_1} = (1+\mu^2)$

ie we have to show that $M_0^2 - 1 = \dfrac{\rho_1 - \rho_0}{\rho_0 - \mu^2\rho_1}$ ie $M_0^2 = \dfrac{(1-\mu^2)\rho_1}{\rho_0 - \mu^2\rho_1}$

using $\rho_0 v_0 = \rho_1 v_1$ ie $\rho_1 = \rho_0 v_0/v_1$, the R.H.S $= \dfrac{v_0(1-\mu^2)}{v_1 - \mu^2 v_0} = \dfrac{v_0^2(1-\mu^2)}{v_1 v_0 - \mu^2 v_0^2} = \dfrac{v_0^2(1-\mu^2)}{c_*^2 - \mu^2 v_0^2}$

$= \dfrac{v_0^2(1-\mu^2)}{c^2(1-\mu^2)}$ using (116) $= M_0^2$ //



$\phi = \phi(P, e)$

(122): $\mu^2 v_0^2 - C_*^2 + (1-\mu^2) c_0^2 = 0$

$\mu^2 v_0^2 - v_0 v_1 + (1-\mu^2) c_0^2 = 0$. Solve for v_1 and here $v = v_0$, $u_p = v_0 - v_1$ } $v_1 = v_0 - u_p < v_0$ since v_0 is supersonic & v_1 " subsonic }.

(46) – 9/3/57 – (i) Derivation of (122) & consequences.

(ii) – Shock compressive – Rarefaction shocks against 2nd Law of Thermodynamics.

Proof sketch: $T d\phi = de + p d\tau$; $P = e + p\tau$; $[dq = de + p d\tau = T d\phi$. 1st Law.

$$\phi_1 - \phi_0 = \int_0^1 \frac{de + p d\tau}{T} = \int \frac{dP - \tau dp}{T}$$

$C_p = \frac{dP}{dT}$, $C_v = \frac{de}{dT}$; $p\tau = RT$.

$$= \int_0^1 \left(C_p \cdot \frac{dT}{T} - R \cdot \frac{dp}{p} \right)$$

$\frac{C_p}{C_v} = \gamma$

$$= C_p \log\left(\frac{T_1}{T_0}\right) - (C_p - C_v) \log\left(\frac{p_1}{p_0}\right)$$

$$= C_p \log\left\{\left(\frac{T_1}{T_0}\right)\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}$$

$$= C_p \log\left\{\left(\frac{p_1 \tau_1}{p_0 \tau_0}\right)\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} = C_p \log\left\{\frac{\tau_1}{\tau_0}\left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}}\right\}$$

& from Hugoniot's relation $= C_p \log\left\{\frac{\tau_1}{\tau_0}\left(\frac{\tau_0 - \mu^2 \tau_1}{\tau_1 - \mu^2 \tau_0}\right)^{\frac{1}{\gamma}}\right\}$

From $\rho_0 v_0 = \rho_1 v_1 \rightarrow \frac{v_0}{\tau_0} = \frac{v_1}{\tau_1}$ $= C_p \log\left\{\frac{v_1}{v_0}\left(\frac{v_0 - \mu^2 v_1}{v_1 - \mu^2 v_0}\right)^{\frac{1}{\gamma}}\right\}$

If $x = \frac{v_1}{v_0}$; expression within $\{\ \} = f(x) = x\left(\frac{1-\mu^2 x}{x - \mu^2}\right)^{\frac{1}{\gamma}}$

It can be shown that if $x < 1$, $f(x) > 1$ i.e. $\phi_1 - \phi_0 > 0$. $\phi_1 > \phi_0$

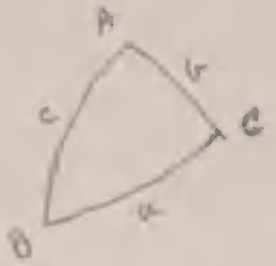
& if $x > 1$, $f(x) < 1$ i.e. $\phi_1 - \phi_0 < 0$, $\phi_1 < \phi_0$.

But $x < 1$, since across the shock supersonic → subsonic i.e. $v_1 < v_0$ i.e. satisfying inequality.

i.e. $x < 1$ is forbidden by 2nd law of Thermodynamics i.e. no expansion shocks.

Syllabus for sighting

General Principles; Parking bar sights; Reciprocating sights; Oscillating sights; ~~Field Artificial~~ Drift Compensation & Compensation for tilt in trunnion; Field, Anti-Tank and Coast artillery sights.



(iii) what happens inside a compression shock – Taking viscosity into consideration, one can get the equation $\frac{8}{3}\frac{\mu}{m(\gamma+1)} v \frac{\partial v}{\partial x} = (v-v_0)(v-v_1)$

or solution is: $\frac{3}{8}\frac{m(\gamma+1)}{\mu} x = \frac{v_0}{v_0-v_1}\log\frac{v_0-v}{v_0-v_1} - \frac{v_1}{v_0-v_1}\log\frac{v-v_1}{v_0-v_1}$

In conservation laws μ does not enter & if $\mu=0$, $v=v_0$ or $v=v_1$ i.e μ is important where v_1 & v_2 differ i.e in transition region which shrinks to zero if $\mu=0$. Thus occurrence of shock is a boundary layer phenomenon inside fluid.

(iv) oblique shocks – continuity of tangential comp. of velocity for non-viscous fluids, $l_0 = l_1 = l$.

$N_0 N_1 = C_*^2 - \mu^2 L$. – $N_1 < N_0$, flow during turned towards shock line – Shock polar.

(v) Detonations & Deflagrations –

11/3/57 – General Lecture on Topology.

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Lecture Notes on Sighting to 8" T.S.O's (31/1/58)

1st Lecture – 03/11/58.

Recapitulation of spherical trigonometry –

(1) $\cos a = \cos b \cos c + \sin b \sin c \cos A$, ..

(2) $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$

(3) Four consecutive elements: tan (middle) cos (inner angle) = sin (inner side) cot (other middle) – sin (inner angle) cot (other angle)

(4) $\left.\begin{aligned}\sin a \cos B &= \cos b \sin c - \sin b \cos c \cos A \\ \sin a \cos C &= \sin b \cos c - \cos b \sin c \cos A\end{aligned}\right\}$

Polar Formulae dual to (1) – (4) – Change a to $180°-A$, A to $180°-a$, etc.

(2) & (3) remain invariant under this

while (1) & (4) give new formulae.

2nd Lecture – 1/2/58 – Remaining portions – formulae for $\nabla^2 L^0 \Delta^S$ – Napiers Rules.

3rd Lecture – 3/2/58 – finished sph. Trigonometry – mentioned also area $= Er^2$ & formula for $\cos\frac{1}{2}E$.

4/2/58 – Did not take a class because I was busy with Karnataka papers..

4th Lecture – 5/2/58 – General Lecture on celestial sphere – alt.-azimuth, (α, δ) (A, ϕ) – Proposed a scheme of colloquium in respect of dealing with the subject.

5th Lecture – 8/2/58 – met the class & arranged that from 10/2/58, Major Bathi & Payara (who have Instruction) should handle the subject).

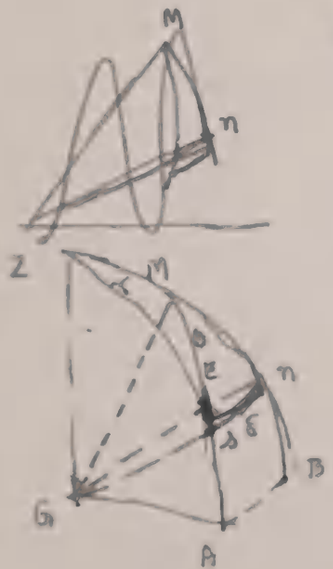
6th Lecture – 10/2/58 – Lecture by Major Payara.

7th Lecture – 11/2/58 – continuation by Payara

8th Lecture – 12/2/58 (i) Derive 8·345, p. 17 of Sighting pamphlet No.3 – note that fig 8·345 (p. 18) GS and GN are ⊥l to MS and MN of fig 8·03 (p. 2); hence do 8·345 taking Gs & Gn ⊥l to MS and MN.

(ii) Also derive, $\phi - E = -\frac{b^2 \sin 2E}{4}$ as in fig 8·3451

(iii) Also show that $\delta = b \sin E$, when angle of sight $\neq 0$ ie 8·346 of p. 19 – fig 8·346 is wrong BMS = b & not ZSM as marked in figure – Draw a better figure than fig 8·346.; Also note that $Z\hat{S}n = 90°$ from def^{n} of latral plane of sight given on p. 19 of pamphlet on Geom. Theory (G.M.6) as being ⊥r to plane of sight.



From Δ Z Sn ($Z\hat{S}n = 90°$)

$\cos S = \tan\delta \cot r$

$\tan r \cos S = \tan\delta$ or $\delta = r \cos S$ .. (i)

AS = S = angle of sight

Sm = E = hayl. el^{n} ha

From Δ Z'MS ($Z\hat{M}S = 180° - \theta$)

$\frac{\cos S}{\sin\theta} = \frac{\sin E}{\sin r}$ or $\theta \sin E = r \cos S$ .. (ii)

$\therefore \quad \delta = \theta \sin E$ .. (iii)

as for case of $S = 0$ as in (i).

9th Lecture – continuation by Major Payara.

(1) 8·05.

(2) Target correction for drift

(3) 8·06.

(4) Reciprocating rocking bar sights.

16th Lecture – 17/2/58 – Major Payara completed his assignment.

11th Lecture – 19/2/58 – Diffce between bubble on oscillating bracket & bubble on rocking bar – Derivation of formulae in the above case // Calibrating sights

Finished this course on Sighting by having lectures by the Sheaf Officers Instructs

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(5) Sighting Systems to 7th T.S.O's started on 23/5/59.

1st Lecture – Introduction – The path ie the trajectory of a projectile depends upon the action of gravity and the (23/3/59) resistance of the air. Gravity causes the trajectory to bend downwards, and if the missile be an oblong projectile rotating about ~~axis~~ its longitudinal axis, the resistance of the air will cause it to bend either to the right or left depending on the direction of rotation of the missile. This bending is called the ~~de~~ deflection due to drift or merely drift. Thus the axis of the gun has to be pointed above and to one side of the actual target desired to be reached by the projectile. This process of pointing the gun in a proper direction and desired angle of elevation is called known as laying of the gun. The mechanical devices employed to facilitate accurate laying constitute the sighting system. Sighting devices are precision devices comparable to a surveyor's transit which measures horizontal and vertical angles.

Fig (i)

Fig (ii)

Definitions – (i) Line of sight ie O T (T is target)

(ii) Tangent elevation is A O T ; (iii) Quadrant elevation is A O H ; (iv) angle of sight is T O H

(v) axis of the sight is line joining the hind (or rear) sight to the foresight & this should be identical with the line of sight.

The processes of laying may be considered under two heads (i) laying for line or direction, (ii) laying for elevation.

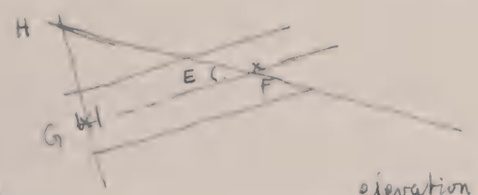
- Two important sighting systems (i) Rocking bar sights (ii) Independent line of sight systems.

In (i) the ~~line~~ axis of ~~sight~~ is set at the appropriate angle to the axis of the bore, and the whole assembly of gun and sights then moved so that the line of sight is on the target.

In (ii) the tangent elevation is set by ~~moving the~~ elevating the gun alone through the required angle, the ~~line~~ axis of sight remaining stationary. The angle of sight is applied through a second elevating gear which moves the gun & the sight together through the angle of sight.

Considering (i) the setting of the line of sight is done by having a movable hindsight, but the foresight fixed on the axis of the bore, and the hind sight is movable on in a socket fixed to the gun so that as in the figure HG is always $\perp^r$ to GF (axis of the bore). on the elevation being ordered H is moved out so that GFH = req^d tangent elevation. Thus

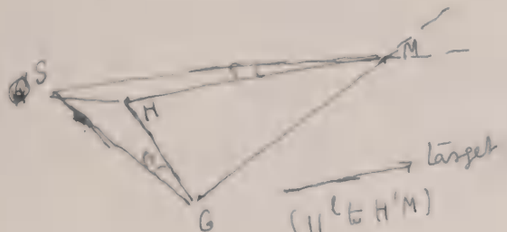
$$HG = GF \cdot \tan E$$ ie HG is graduated $\propto \tan E$



Next HF is brought on to the target & the whole assembly moved

(2nd 1) This is for laying for elevation. As regards laying for line or direction, let us suppose that GH while remaining $\perp^r$ to axis of the bore is slightly tilted to the left so as to make a small angle with the vertical plane of departure, and then H adjusted so that SM is the line of the sight directed on to the target, and we will consider here the particular case where the angle of sight is zero ie SM is horizontal (or target in horizontal line)

(24/3).



Since SM is along the target, the gun points to the left of the target. The angle SMH is the deviation in the angle sight & is the drift δ. The angle HGS is the tilt θ & G$\hat{M}$H is the tangent elevation E. We could call here the angle GMS as the quadrant elevation E'. One has to find δ in terms of θ & E. Since this requires use of a good bit of solid geometry it is preferable to use spherical trigonometry (where more than two directions are involved).

Spherical Trigonometry :- Small circles & great ○^{s} - Pole & polar - fund ○^{le} & secondaries, λ & ϕ - Angle between two great ○^{s} / Sph. Δ^{le}, sides & angles - Principle of duality.

(3rd Le)

3rd Lecture (31/3/59) – Sph. Δles; sides & angles; convention re. lengths of sides leading to any angle being less than π – Polar triangle; polar of polar Δle; $A' = \pi - a$, $a' = \pi - A$; Principle of duality – Elementary geometrical theorems; $a+b > c$; $a+b+c < 2\pi$; $2\pi < A+B+C < 3\pi$; Congruence & symmetrical equality of Sph. Δles, Test conditions for congruence; If $A = B$, $a = b$.

4th Lecture (2/4/59) – Derivation of $\cos a = \cos b \cos c + \sin b \sin c \cos A$ & its dual – also $\cos A = \dfrac{\cos a - \cos b \cos c}{\sin b \sin c}$ and its dual – Derivation of $\dfrac{\sin A}{\sin a} = \dfrac{\sin B}{\sin b} = \dfrac{\sin C}{\sin c} = \dfrac{2\pi}{\sin a \sin b \sin c}$

& $\dfrac{\sin a}{\sin A} = \cdot = \cdot = \dfrac{2N}{\sin A \sin B \sin C}$

– Statement of the rule: $\cos$ (inner side) $\cot$ (inner angle) = $\sin$ (inner side) $\cot$ (other side) – $\sin$ (inner angle) $\cot$ (other angle).

5th Lecture – (3/4/59) – Formulae for rt $\angle^d$ Sph. Δles – $C = 90°$ – Napier's scheme & Napier's rules giving the ten formulae – Proof of these formulae – Examples on Sph. Trig:

6th Lecture (4/4/59)

(1) In an equilateral spherical triangle prove that $\sec A = 1 + \sec a$. [one fund. formula]

(2) Two ports are in the same parallel of latitude, their common latitude being l, and their diffce of longitude is 2λ. Show that the saving of distance in sailing from one to the other on the great Δle instead of sailing due East or West is

$$2r\left\{\lambda \cos l - \sin^{-1}(\sin\lambda \cos l)\right\}$$

(3) If a ship be proceeding uniformly along a great Δle and the observed latitudes be l_1, l_2, l_3 at equal intervals of time in each of which the distance traversed is s, show that

$$s = r \cos^{-1}\left\{\frac{\sin\tfrac{1}{2}(l_1+l_3)\cos\tfrac{1}{2}(l_1-l_3)}{\sin l_2}\right\}$$

(4) In a right-angled Δle if δ be the length of the arc drawn from C to the hypotenuse AB, show that $\cot^2\delta = \cot^2 a + \cot^2 b$. [If $\angle^r$ divides C into angles $\alpha + \beta$, $\cos\alpha \cos\delta = \sin\delta \cos b$ & $\cos\beta \cos\delta = \sin\delta \cos a$. Square & add.]

(5) A ship starts from a point on the equator and sails in a great circle, cutting the equator at an angle of 45°. Find how much she has changed her longitude when she has reached a latitude $\tan^{-1}\left(\frac{1}{2}\right)$. [From Rt $\angle^d$ Δ formulae $\sin A = \tan b \cot c$, etc.]

(3/4)

(6) If two sides of a sph. Δle be supplementary, prove that the meridian passing through their intersection is a quadrant.

(7) Prove that the area of a spherical triangle is equal to $E r^2$, where E is the spherical excess.

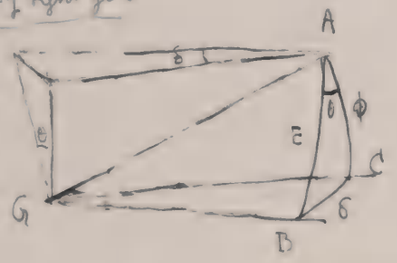
(8) Legendre's theorem – If the sides of a spherical triangle are small compared with the radius of the sphere, then each angle of the sph. Δle exceeds by $\frac{1}{3}E$ the corresponding angle of the plane triangle whose sides are of the same lengths as the arcs of the sph. Δle – without proof –

(9) The area of a sph. Δle formed on the Earth's surface being known in sq. ft, find a rule for computing the spherical excess in seconds – General Roy's rule

(10) Geodetic surveying – In forging the greatest Δles of English survey $E > 30''$ & max. reached is $64''$

7th Lecture (6/4/59) – Gave above examples (1), (2), (3) & (6) for homework – Drift correction with the rocking bar right when the angle of sight is zero and small difference between tangent and quadrant elevations – Drift correction in the general case when the angle of sight is not zero also being the same ie deflection is independent of angle of sight.

(i) Angle of Sight zero

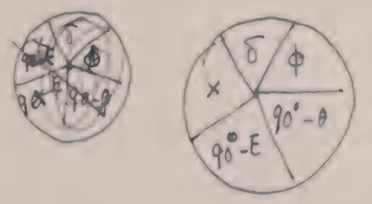


In the accompanying figure ϕ = quadrant elevation is above horizontal

& E = tangent elevation is above line of sight

The E of § 8·03 is ϕ here
E' " is E "

$\hat{C} = 90°$ because AGC is vertical & BGC is horizontal



$$\sin\delta = \sin\theta \sin E$$

$$\text{or } \delta = \theta \sin E.$$

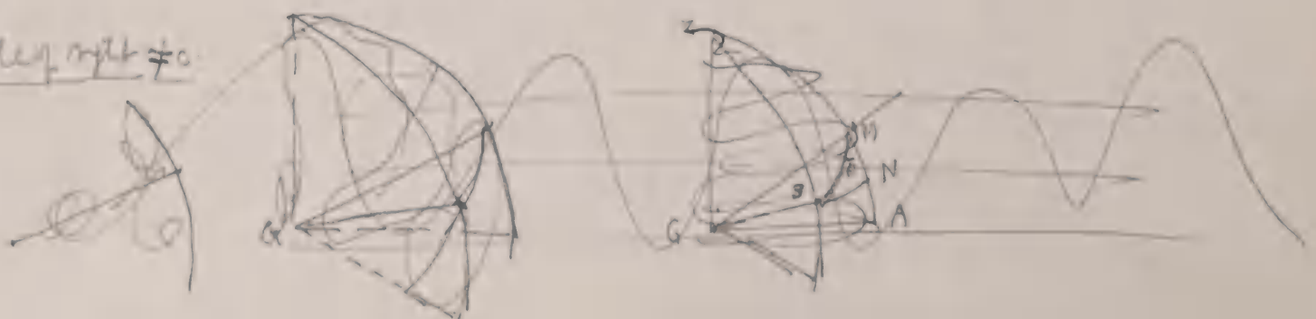
$$\cos\theta = \tan\phi \cot E \quad \text{or } \tan\phi = \cos\theta \tan E \cong \tan E\left(1 - \frac{\theta^2}{2}\right)$$

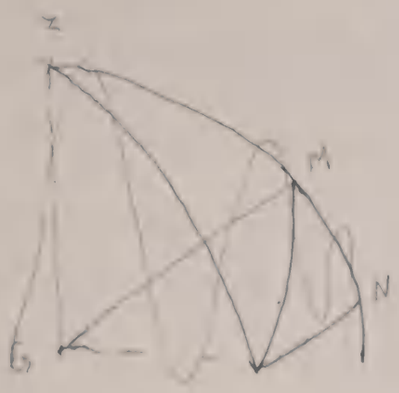
$$\frac{\sin\phi}{\cos\phi} = \frac{\sin E}{\cos E}\left(1 - \theta^2/2\right) \therefore \frac{\sin E}{\cos E}\cdot\frac{\theta^2}{2} \cong \frac{\sin E}{\cos E} - \frac{\sin\phi}{\cos\phi} = \frac{\sin(E-\phi)}{\cos E\cos\phi}$$

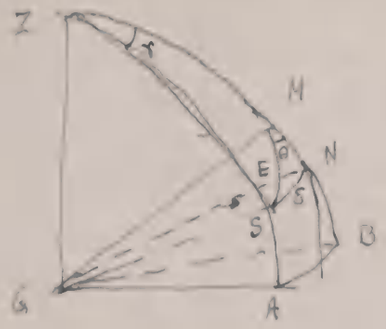
$$\sin(E-\phi) \cong \frac{\theta^2}{2}\cdot\sin E\cos\phi \quad \text{or } \phi - E \cong -\frac{\theta^2}{2}\sin E\cos E \cong -\tfrac{1}{4}\theta^2\sin 2E$$

With $\theta = 2°$, $E = 45°$ (Max. range), this gives $\phi - E = 1.5'$ which is negligible.

(ii) Any angle of sight $\neq 0$







The essential point is that the plane SGN is L^r to the vertical plane through GS i.e. $\hat{ZSN} = 90°$.

From ΔZSN ⟶

$\sin ZS = \tan\delta \cot r$

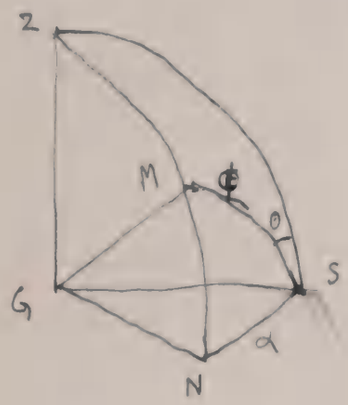
or $\delta = r \sin ZS$ ——(1)

From ΔMSZ (not rt-L^r), $\frac{\sin r}{\sin E} = \frac{\sin\theta}{\sin ZS}$

$r \sin ZS = \theta \sin E$ —— (2)

Δ from (1) & (2), $\delta = \theta \sin E$.

(iii) Correction for tilt of mounting – trunnion axis not being horizontal but inclined at a small θ to the horizontal. Let GZS be the originally vertical. If the figure be elevated through E to GM, GMS will be not be in the vertical plane but inclined to vertical at angle θ, so that an automatic drift α is introduced. From Δ^{le} MNS rt-L^r at N., Napiers



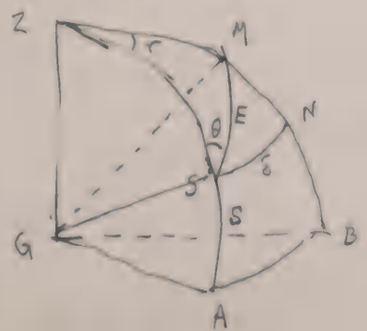
scheme is

$\sin\theta = \tan\alpha \cot\phi$ or $\tan\alpha = \sin\theta \tan\phi$

$\alpha = \theta \tan\phi$.

8th Lecture (8/4/59): (a) A tilt A simple reciprocating sight is one in which compensation is automatically made for the tilt of trunnions & also for wheels of carriage not being level. In a simple reciprocating sight correction for drift can be obtained in two ways :- (i) as in the ordinary sight with $\delta = \theta \sin E$, whether angle of sight be zero or not; (ii) by over or under compensation for tilt of trunnions ~~gives~~ $\delta =$ giving $\delta = \theta \tan E$ for angle of sight zero (really $\theta \tan\phi$ but $\phi \simeq E$), and for angle of sight equal to S giving

$$\delta = \frac{\theta \tan E}{1 - \tan E \tan S}$$ – Proof of this – The figure should be that of 8·346 with $\theta = ZSM$, in fact in the book the figures 8·346 & 8·348 have been interchanged.



On adjoining figure $\hat{ZSM} = \theta$ & as before $\delta = r \sin ZS$ or $\delta = r \sin S$

From ΔZSM, $\frac{\sin r}{\sin E} = \frac{\sin\theta}{\sin ZM}$ or $r = \theta \cdot \frac{\sin E}{\sin ZM} = \theta \cdot \frac{\sin E}{\cos BM}$

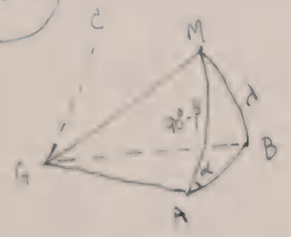
$\Delta\ BM \simeq BN + NM \simeq S + \phi \simeq S + E$

$\therefore\ r = \frac{\theta \sin E}{\cos(E+S)}$ = ~~$\theta \sin E$~~ & $\delta = \frac{\theta \sin E \cos S}{\cos(E+S)}$

$$\delta = \frac{\theta \tan E}{1 - \tan E \tan S} \quad \text{(as in 8·11)}.$$

(b) Case of a non-reciprocating ~~sight~~ right in which an error is made in angle of right:

§ 8·21



Let GM be the direction of shooting of the gun being in a plane inclined at an angle γ to the horizontal, α being error in angle of right & let GM make an angle β with the line of greatest slope on the plane i.e. $MGA = 90° - \beta$. Let inclination of GM to horizontal be λ i.e. in rt^{L} $\angle$d $\triangle$ AGM ($B = 90°$), $AM = 90° - \beta$, $MB = \lambda$, $\hat{A} = \gamma$

Hence from Napier's rule

$\cos \alpha \sin \lambda = \cos \beta \cos \alpha$ or ~~$\lambda = \cos \beta$~~ or $\lambda = \alpha \cos \beta$

The trunnion axis is $\perp^{r}$ to GM, hence tilt in trunnion axis is $\lambda = \alpha \cos (90° - \beta)$
$= \alpha \sin \beta$.

∴ If the angle of right be assumed zero, drift is given by

$$\delta = \lambda \tan \phi = \alpha \sin \beta \tan \phi.$$

.. .

(A) The fundamental particles by Clifford E. Swartz (1964)

(1) Building blocks & classification schemes

(a) Prologue – Universe on log. scale – Chart with 0 = 1 cm, 2 = 1 meter, 6 = 10^6 cm, −2 = 10^{-2} cm etc – Tree man 2, Dinosaur 3.5, Earth 9, Earth's orbit 13, Nearest star 18, Galaxy 23, Edge of visible Universe 28 – Bacteria (−3), λ for visible light (−6), λ of 1 Mev x-ray (−10), nucleons (−13) – Mendeleeff's periodic table

(b) What is a fundamental particle? Protons, Neutrons and electrons are building blocks of the atoms and thus of all the elements, but the question that they are fundamental is different. No particular expt. demands independent existence of electrons in the atom. It is the successful model of the atom which gives them such existence. When energy is applied to matter in the proper way, electrons are emitted from all substances. For that matter so is light, yet we do not think of light as being an independent constituent of the atoms. For eg. in β-decay of radioactive materials electrons are emitted from the atomic nucleus, but the model insists that the electrons were not in the nucleii to begin with but were created on the spur of the moment. This interpretation is considered as the "truth" because the model is so successful in correlating a vast number of experimental facts – On the analogy of the H-atom existing in different states, most particles investigated cannot be considered fundamental; within several family groupings they change into each other with ease. Even before their transient existence before decay, particles may be considered to be partially themselves and partially other systems with which they are related. The heavier particles (including p and n) may not even have a core identity of their own; perhaps each exists only because of and in terms of interactions with all the others.

Even protons & neutrons cannot be considered fundamental & certainly not elementary. For eg. the neutron is radioactive. When free of an atomic nucleus, it decays into a proton, electron and neutrino, although no theory views it as composed of these particles. Since 1958 the internal structure of both p and n has been probed with high energy particles. For present purposes any attempt to define what we mean by "fundamental" must be abandoned.

The view that an object is real only if it can be seen is also meaningless & should be given up. The signals given by cloud chambers, etc, etc. give recognizable patterns as real as objects actually seen, and perhaps more effectively. Also the proton for eg. is much simpler, and therefore we can almost see it far more completely than we ever can see a human. Analogy of recognizing a friend by dialling his phone number.

(2) Bohr atom (a) Bohr atom models of Hydrogen, Deuterium (heavy hydrogen), and Carbon – out of style since 35 years (i.e. at 1964) – nothing of discrete distances of electrons and shells. Some useful features are still valid.

(2) (a) Size and Mass — Size $\sim 10^{-8}$ cm for each atom (from H to U size never varies by more than a factor of 2) on a cm. cube the no. of atoms would be $(10^8)^3 = 10^{24}$, and because of the factor 2 in linear size, this number is close to Avogadro's number (i.e. no. of molecules in any gram-molecular weight of a gas at NTP $\cong 6 \times 10^{23}$); Molecular weight = wt of molecule in terms of wt of a H-atom; gram-molecular weight is the number of grams of the gas numerically equivalent to the molecular weight). Avogadro's number ($\cong 6 \times 10^{23}$) is sometimes called the "mole". A mole of atoms has a mass in grams equal to the number called the atomic weight. A mole of H atoms has a mass of 1gm, of deuterium 2gm, of U about 238 gm. Since most of the mass resides in the nucleus and since $\sim 10^{24}$ H-atoms weigh 1gm, then each atom of H & hence each proton must weigh 10^{-24} gm. The neutron must weigh nearly the same.

(3) (b) valence electrons — Electrons exist in shells with only the outermost taking part in chemical combinations

(d) Isotopes — having same no. of protons & electrons (hence same chemical properties) but with varying number of neutrons in the nucleus. For eg. H has 3 isotopes viz $_1H^1$, $_1D^2$, $_1T^3$ with nuclei consisting of one proton, one proton & one neutron, one proton and 2 neutrons respectively.

(4) (e) Periodic table — nature determined by arrangement of electrons in the orbits. The $-^{ve}$ electrons are pushed towards the $+^{ve}$ nucleus & take up orbits satisfying some laws, the most important of which is the exclusion principle — closed shells of electrons & additional electrons occupying unfilled orbits at greater distances from the nucleus — The periodicity in Mendeleeff's table arises because an atom with one valence electron outside first closed shell (alkalies: $_3Li^6$) acts chemically like the much heavier atom (alkalies) with one valence electron outside its second closed shell ($_{11}Na^{23}$). [$_3Li^6$ has 3 protons & 3 n; [# of n.p.?]]

(5) (f) Binding energy — Energy binding electrons to nucleus & if v_e be esc escape velocity $E = \frac{1}{2} m v_e^2$, for outermost electrons this is just equal to energy involved in chemical reactions. A convenient unit of energy for this type of reaction is an electron volt i.e. one electron charge raised one volt in potential (~ energy per molecule in chem. reactions) and is the basic unit for the microworld. for the macroworld the joule (i.e the watt-second) of energy is useful. one joule = 6×10^{18} eV. Mev, Bev (for 10^6 & 10^9 eV). Electric charge is quantized & the energy unit might have been called "charge volt" or "proton volt".

(7) (g) Nucleus — Diameter of nucleus smaller than that of the atom by a factor of 10^4 to 10^6 (If nucleus of H is as large as a child's marble, the electron would be at a distance of 1/8 th of a mile) — Hard-sphere model's radius given by $r = r_0 A^{1/3}$ (r_0 the radius of D or $\frac{1}{2}$ $\cong 1.3 \times 10^{-13}$ cm) and

A is the number of nucleons.

(C) (@) The electron, a familiar particle. — J.J. Thomson's expt of 1897 of deflecting path of cathode rays by uniform electric and magnetic fields.

(1) (@) Production — Electron is easiest particle to isolate — To drive an electron out of a material, it is necessary to give it sufficient energy to escape from its atomic or molecular bonds — Thermal energy or energy by a photon i.e. photoelectric effect; in former case for eg. metals can be heated & the electrons boiled off; in the latter photon energy $E = h\nu = E_{escape} + \frac{1}{2}mv^2$ & in former case $E_{thermal} = E_{escape} + \frac{1}{2}mv^2$ — Energy for escape is a few eV; ordinary blue light providing about 3 eV is sufficient for electron emission from alkali metals; ultraviolet energy of about 6 to 10 eV called knock out electrons out of other materials under suitable conditions — Thermal distn of energy for gas molecules similar to that of average thermal energy; eg. of Tungsten filament at T = 3000° Kelvin (10 times as hot as the 300° K room temp) — Simple diode to demonstrate negative current emission from tungsten filament in vacuum connected with battery to heat element, meter to read current & plate receiving plate — Millikan's oil drop expt was the first (1903) to measure the individual charge of electron, which gave the charge $-e$ & equal to 1.6×10^{-19} coulomb, & a coulomb is the amount of charge transferred by a current of one ampere flowing for one second; in terms of the fundamental electron charge, one ampere is the flow of $\frac{1}{1.6 \times 10^{-19}} = 6 \times 10^{18}$ electrons passing through a given point each second.

(2) (@) Mass — Charge is the most obvious attribute of electron; Next comes question of its mass — Two methods of measuring mass (i) using $F = G \cdot mM/r^2$ & (ii) $F = ma$ · (i), is completely impractical since F is too small & (ii) can be used by bubble chamber expts. to find the acc^n a of the particle which is given by $\frac{Bev}{mv}$, where B = strength of magnetic field, e is electron charge & v its velocity using centripetal acceleration v^2/r & thus getting $Bev = mv^2/r$ & solving for m, $m_{electron} = \frac{Ber}{v}$ & this turns out to be 9×10^{-31} Kg $= 9 \times 10^{-28}$ gm $\cong 10^{-27}$ gm [Cf. with m_p or $m_n \cong 10^{-24}$ gm]

[Here the author uses Bev for the L^r force in a magnetic field is confusing since Bev stands for a billion electron volts as mentioned on p. 8. Instead of B, H might have been used here. This might be communicated to the author as a trivial error to be corrected in later editions]

Another method of measuring m_e — creation of an electron and positron by a photon & the pair made to circle in opposite directions in a magnetic field (the photon given out by an x-ray of at least 1 Mev necessary for creation of the pair and each particle of the pair uses half of this 1 Mev for its mass) and this method also gives the energy mass equivalent of mass as 0.51 Mev

[Here again in the line above, the arithmetic, the author makes the mistake of saying that

"that the mass is 0.51 Mev"; it should be "that the energy equivalent of the mass is 0.51 Mev" [[The author might be written to about this]] - Arithmetic that follows shows that with $m = 9 \times 10^{-31}$ kg, $mc^2 = 81 \times 10^{-15}$ J and since 1.6×10^{-19} Joules = 1 eV, $mc^2 = 0.51 \times 10^6$ eV = $\frac{1}{2}$ Mev.

(3) (5) Spin - three effects:

(i) mag. moment (i) A spinning electric charge becomes a small magnet - orientation in uniform magnetic fields - in an inhomogeneous magnetic field use - Stern-Gerlach expt. of 1924 using the valence electron of silver (instead of bare electrons); figure showing expt. methods and techniques - In any expt. arrangement the electron where electron spin is determined some standard direction or axis is established. The electrons are always found to be lined up either in that particular direction or opposite to it, but never in between (To assume random orientations before, but arranging themselves in this way when entering the magnetic field is metaphysics & meaningless)

Unit of spin (or A.M.) is $\hbar = h/2\pi = 1 \times 10^{-34}$ joule-sec and of dimension (ml^2/t) - Striking that there should be a natural unit of A.M. in this world, while there are none for length, time or mass. Electrical charge has a natural unit & now we have A.M. The electron has spin 1/2. Like E spin is quantized & can exist only in multiples of $\frac{1}{2}\hbar$ (never with values in between like $0.65\hbar$). Also measured values corresponding to different directions of the spin must always differ from each other by one whole unit $\hbar$; the valence electron of silver is found in only two orientations, with direction - giving field $(+\frac{1}{2})$ or against $(-\frac{1}{2})$

(ii) magnetic moment effect of electron spin can also be determined in uniform magnetic fields, unlike a in inhomogeneous magnetic field of the Stern-Gerlach expt. - use of substances containing unpaired electrons which do not have to interact with the neighbouring atoms to form molecular bonds - The rest of the explanation makes no meaning & there appears to be no coherent explanation of the experiment described which merely refers to "absorption of radar frequency energy due to electron magnetic resonance.

X ||

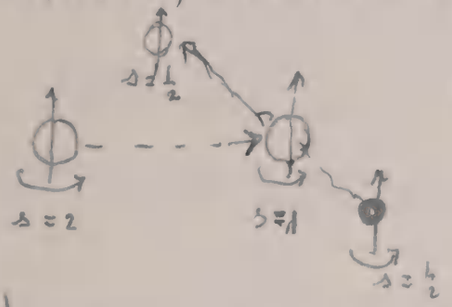
(iii) Different energy states of electron as it assumes the two orientations (in or out of the magnetic field) give rise to spectroscopic effects. This was known 100 years ago, but only in 1924 Goudsmit and Uhlenbeck gave the correct interpretation as due to the electron spin.

X || splitting of spectral lines.

(4) (ii) angular momentum, a second effect of spin - Electron spin also to be taken into account in spectroscopy and in particle delays where electrons are emitted because A.M. is one of the things uniformly conserved. Both these due to fact that photon has spin, nucleus has spin, and all spins must be added [illegible]

ie many caneal · The spectral lines can be understood only when the description of total A·M · includes the spin

In particle decays also, the same conservation principle must be satisfied · for eg · when the muon (one of the main parts of cosmic rays at sea level) decays, a single electron comes spinning out · In this case, as in many other cases, at least one other particle the neutrino ν is also emitted (Neutrino has no charge or mass but only spin)



Picture of a nucleus with spin 2 $\hbar$ decaying to nucleus with spin $\hbar$ plus electron and neutrino with spin $\frac{1}{2}\hbar$ each.

(iii)

(iii) The Exclusion law, the third role of spin — Pauli's principle — fermions — Exclusion Law : No two fermions can possess identical properties while inhabiting the same region. This restriction is responsible for the particular arrangement of electrons in atoms and for determining the nature of the periodic table —

Figure showing the Pauli exclusion principle and the periodicity of the Bohr orbit occupation for H, He, Li, Be, B, C, N, O, F and Ne (ie up to ~~no of e = 2~~, $n = 1,\ l = 0,\ s = \pm \frac{1}{2}$

$n = 2,\ l = 0,\ s = \pm \frac{1}{2}$

$l = 1\ (\uparrow),\ s = \pm \frac{1}{2}$

$l = 1\ (\rightarrow),\ s = \pm \frac{1}{2}$

$l = 1\ (\uparrow),\ s = \pm \frac{1}{2}$

(iv)

(iv) Size of the electron - wave particle problem : - Relation between size and the particle - wave duality for eg. using $\lambda = h/p$ $\Delta p = \dfrac{m_0 v}{\sqrt{1-(v^2/c^2)}}$ for $v \simeq c$ (relativistic), λ may be $\simeq 5 \times 10^{-10}$ meter ie larger than size of an atom —

(4)

Wave behaviour of electrons (not wave nature) is brought out clearly by experiments ie. interference of diffraction patterns (just like light) — No need to concern ourselves as to whether an electron is a wave or a particle; an electron is an electron — Exact ~~solns~~ mathematical solns to most physical problems is never possible and part of the skill consists in choosing the right approximation, and knowing to what accuracy the soln can be carried — Eg. of diffl eqn $mg - k\left(\frac{dy}{dt}\right)^2 = m\frac{d^2y}{dt^2}$ (two solns under diff. approxn) — In quantum electrodynamics, when particles are interacting with low energy, the math · description is usually best approximated with eqns like wave eqns of classical mechanics & then quantum mechanics is spoken of as "wave mechanics". On the other hand, the motion of high energy particles can be best approximated by using eqns for motion of a particle. In both cases, the behaviour is being studied and the mathematics gives the probability of finding the particle at a particular place with particular properties. Mathematics for particles ~~gives~~ is very definite about this probability; wave mechanics gives relative probabilities by use of "probability wave".

The apparent duality is a perfectly normal state of affairs in this world. A woman is a woman is a woman. Yet in some circumstances she behaves like an animal and happily, in others, like an angel. (We might have used "angle" and "angel").

? purple?
X

(5) Relativistic effects – Particle reactions occur at velocities very close to c & hence rel. mechanics is the natural tool for describing interactions. Relations:

(i) $E_0 = m_0 c^2 \rightarrow m_{0e} = 0.51$ Mev, $M_{0p} = 1$ Bev, $M_{(1gm\ H_2O)} = 9 \times 10^{13}$ joules = output of Niagara falls for 10 hrs

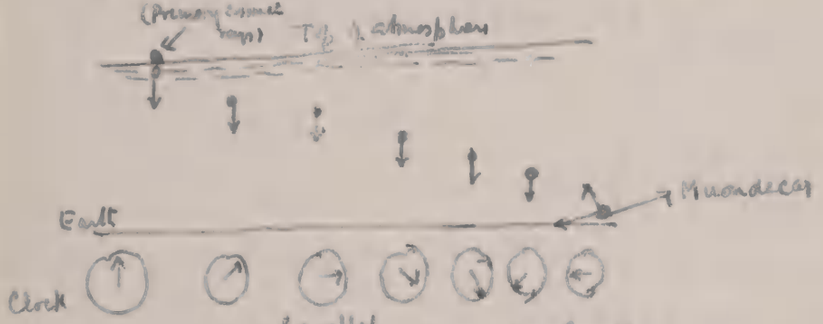
(ii) $m = \dfrac{m_0}{\sqrt{(1 - v^2/c^2)}} = \dfrac{m_0}{\sqrt{1-\beta^2}}$ $(\beta = v/c)$ – manifold exptl. evidence for this.

(iii) $E = mc^2 = \dfrac{m_0 c^2}{\sqrt{(1-\beta^2)}}$ $(E = \text{rest energy} + \text{K.E})$. Expressing $(1-\beta^2)^{-\frac{1}{2}}$ as binomial series

$E = m_0 c^2 + \frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 v^2 \cdot v^2/c^2 + \ldots$ (higher terms negligible for ord. velocities)

(iv) $\Delta t = \dfrac{\Delta t_0}{\sqrt{(1-\beta^2)}}$. ($\Delta t_0$ = time interval for the object's rest frame, Δt = observed time interval

Relativistic dilation of time is seldom a problem with electron reactions, but is of vital importance for phenomena involving slow particles for eg. slow mesons (as seen in fig. below)



Lifetime & distance travelled as seen from the Earth

Lifetime and distance travelled as seen by the muon

[Relativistic dilatation of time for muon decay. In the earth's reference frame the muon travels 20 miles and lives abnormally long. In the muon reference frame, the muon decays in its normal short time, but, to the earth, the distance it travels appears very short.]

Particles which would normally decay in a couple of millionths of a second may last hundreds of times as long. In the above figure for eg. if the clocks of the fast muon had not been slowed by their high speed & its high velocity, it would have decayed 20 miles or so up near the top of the atmosphere, not far from where they were created. The time dilatation does not depend on the particle. It only happens to a meson or a man, but as can be seen from the energy formula there is considerable less energy involved in accelerating a particle than a man. There would seem to be no immediate prospect of providing a ripe young age for an aging astronaut.

(6) Heisenberg's uncertainty Principle – (i) $\Delta x \cdot \Delta p_x \geq h \simeq 1 \times 10^{-34}$ joule, which follows from $p = h/\lambda$

If h were $= 0$, both x & p_x could be measured with ∞-precision simultaneously. But this world would not be. This would at all in this case. (ii) $\Delta\theta \times \Delta L \geq h$ $(L = A.M)$, (iii) $\Delta E \times \Delta t \geq h$ – Measurement of the energy of excited states is done by letting the system return to normal & see how much energy [illegible], but this involves the time of return to normal. The shorter this duration, the more [illegible]

overtones that come up with the burst of oscillations due to emission of photons. In case of visible light $\Delta t = 10^{-8}$ Sec, we have $\Delta E \times \Delta t \geq \hbar \approx 10^{-34}$ j-Sec & $\Delta E \approx 10^{-26}$ joule $\approx 10^{-7}$ Bev which appears a small uncertainty. But when we deal with excited states with life-times as small as 10^{-22} Sec, we have $\Delta E \approx 10^{-12}$ joule ≈ 10 Mev. Uncertainties of this large size can play dominant roles in certain particle reactions.

(D) Chart of particles stable against decay through nuclear forces.

(1) Organisation and familiar features – Chart with historical order of discovery of particles other than the more recent discoveries of particles which decay in times less than 10^{-22} Sec. These are dealt with in Section (H) – Electron, photon, proton, neutron, neutrino, positron, muon, Pi-meson, K, Λ^0, Σ, Ξ^-, η_0 (neutrino), Ξ^0, Ω^-, Resonance particles – Arguments, using Heisenberg's uncertainty principles, to show that electrons cannot be confined in something as small as an atomic nucleus; such an electron would have an uncertainty in energy far greater than the nuclear binding energy (ie nearly 20 Mev as against 8 Mev): If this electron were forced to have small enough momentum to be bound in the nucleus, its position would be uncertain by an amount far greater than the nuclear size. – Next two pages devoted to the chart of the particles under the four heads: Baryons (strongly interacting fermions), Mesons (Strongly interacting bosons, $s = 0$), Leptons (weakly interacting fermions), massless bosons (photon $s = 1$, graviton $s = 2$), and with classification under each heading giving the particles, antiparticles, Rest mass, Half life in Secs, and Decay Schemes Column-wise – Quite a complicated chart difficult to remember in full [Mention is made of Omega hyperon with spin $3/2$, why not use my algebra for this ?] [Also for graviton spin is given 2 & eqn in my algebra could be used] – Historical statements about discovery of particles – The chart on pp. 22–23 is a classification scheme, far more than a list, but less than a theory – Attributes or parameters of various particles

(2) Parameters:

(a) mass – listed naturally according to mass (for Baryons including p & e) from 1476 of Ω to 1115 of Λ, 939.5 of n, 938.2 of P; 548 of η-meson, 494 of K^+-meson & K^- 498 K_1^0 & K_2^0, 140, 135, 140 for π^-, π^0, π^+-mesons resply; 105.7 for μ^- & μ^+ (para-meson or meson), 0.51 for $e^\pm$, 0 for neutrino-muon & neutrino-electron & their antis denoted by ν_μ & $\bar{\nu}_\mu$, ν_e & $\bar{\nu}_e$ (thus neutrinos associated with μ's are different from those associated with electrons ?); 0 for photon & graviton. – for the familiar trio of e, P, n, $m_0 = \frac{1}{2}$ Mev, 1 Bev, 1 Bev. approx)

(b) Particles, antiparticles – Antiparticles mentioned having identical mass & spin, but opposite electric charge

and strangeness number) [Is this correct? yes if we choose opposite λ 0 to be 0, for in case of η^0 and $\bar{\eta}^0$, no. only π^0 and $\bar{\pi}^0$ have vhp. magnetic moments of spins.

the formula $Q = e(I_3 + \frac{b}{2} + \frac{S}{2})$ gives $Q = 0$ for π^0 and $\bar{\pi}^0$, with values $I_3 = -\frac{1}{2}, +\frac{1}{2}$ for

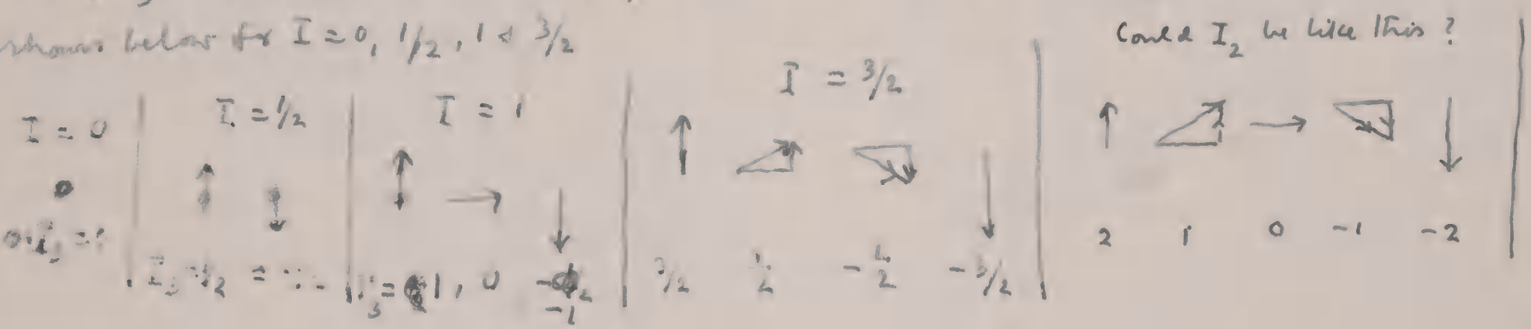
wrong π^0 and $\bar{\pi}^0$ [check on pp. 32-33], $b = +1$ for baryons, $b = -1$ for anti-baryons, $S = 0$ both for π^0 and $\bar{\pi}^0$.

Again for Ξ^0, $I_3 = +\frac{1}{2}$, $b = 1$, $Q = 0$ giving $S = -2$ & for $\bar{\Xi}^0$, $I_3 = -\frac{1}{2}$, $b = -1$ giving $S = 2$. Again for Σ^0, $I_3 = 0$, $b = 1$, $Q = 0$ giving $S = -1$, while for $\bar{\Sigma}^0$, $I_3 = 0$, $b = -1$, $Q = 0$ going $S = 1$. Antiparticles are just as stable (unstable) as their counterparts & so it is therefore preferable to say different I_3's i.e vertical components in isotopic 2-spin. space and so far as we know could form a complete anti universe; perhaps some galaxies may be composed of antimatter — neutrons & antineutrons may seem identical (with $Q = 0$ & $S = 0$), but the distinction is clear, however, in high energy expts where they are produced and detected — Motion of Plate III showing annihilation between p and $\bar{p}$; inverse process of creation of p & $\bar{p}$ possible if enough energy be supplied — for creation of e^- & e^+ each a little more than one MeV recall Plate I shows such creation by a γ-ray; energy > 1 MeV goes into creation + k.E — Three of the particles serve as their own antiparticles. These are the photon, the π^0-meson and the η^0-meson all with zero electric charge and zero strangeness [which have mentioned I_3 also $= 0$ as seen from $Q = e(I_3 + \frac{b}{2} + \frac{S}{2})$ since $Q = 0$, $b = 0$, $S = 0 \rightarrow I_3 = 0$, unlike in the case of η^0 and $\bar{\pi}^0$ where also $R = 0$ & $S = 0$].

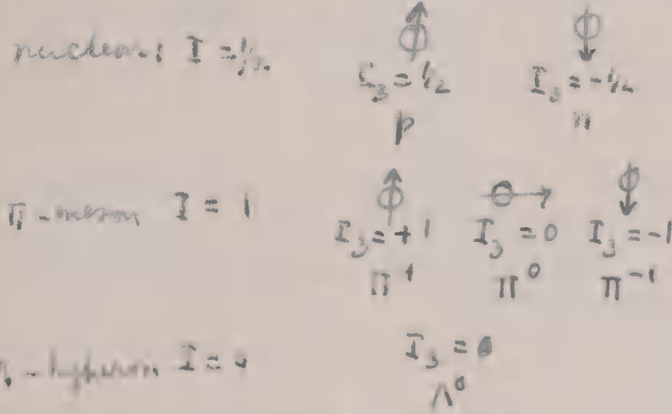
(c) Spin and statistics — Fermions are subject to a conservation principle; they can be created or destroyed only in conjunction with an antiparticle from their own class, for eg. when an electron is emitted from a nucleus (virtual as in β-decay) it is created out of superfluous energy but must be accompanied by an anti-neutrino (to explain surplus energy & to be an antiparticle of the same class) [This conservation is not at all clear & the two counter-examples cited are also not clear] — The above conservation principle does not extend between the two classes of fermions viz the baryons & the leptons. Total no. of baryons remains (such, & so apparently also the total no. of leptons — No such conservation principle applies to bosons. So far as charge, A.M. etc (?) are conserved, bosons can be created without regard to their antiparticles — fermions obey the F-D, & the bosons the B-E statistics.

(d) Isotopic spin: — Notion introduced to fit members of each of the four groups & arrange them so as to display similarities. Also based on the notion of isotopes in case of chemical elements as for example whose atoms have, for eg., the same chemical properties, but different no. of neutrons in the nuclei — The notion is to devise a convenient scheme of mathematical classification — for eg. p and n could be thought of as doublets of the nucleon; alternatively the Λ^0 would be a singlet, while hyperon Σ (Sigma) is a triplet, Ξ (Xi — or cascade-particle) is a doublet & the heaviest of all Ω^- is a singlet. — Classification of isotopic multiplets has an analogy in describing spin of a particle, for eg. for a particle of

of spin S, there can be $(2S+1)$ possible states for eg. $S=0$, this one state, $S=\frac{1}{2}$ gives 2 states $+\frac{1}{2}, -\frac{1}{2}$, $S=1 \rightarrow 1, 0, -1$, and so on ie there is no way way for a particle to change its spin orientation except in unit amounts; if for eg. an atom gives off a photon of light in optical emission, there is a change of one unit since the photon has spin 1, or again when an unstable particle emits a β-decay it loses a spin $\frac{1}{2}$, but because leptons are conserved there must be another spin loss of $\frac{1}{2}$ due to a neutrino, leading to a change of 0 or 1 — Some of a mathematical nature has been used to label the particle multiplets by assigning an isotopic spin number I — This $I=0$ for singlets, $=\frac{1}{2}$ for doublets, $=1$ for triplets and so on. But assigning particular values to members of a multiplet as though they were components of a vector in isotopic spin space takes on a profound aspect in complicated reactions. Thus if I_3 were the vertical component of I, the orientation of I so that the I_3's differ only by ± 1 are shown below for $I=0, \frac{1}{2}, 1 \& \frac{3}{2}$



The counterparts of these in actual cases are



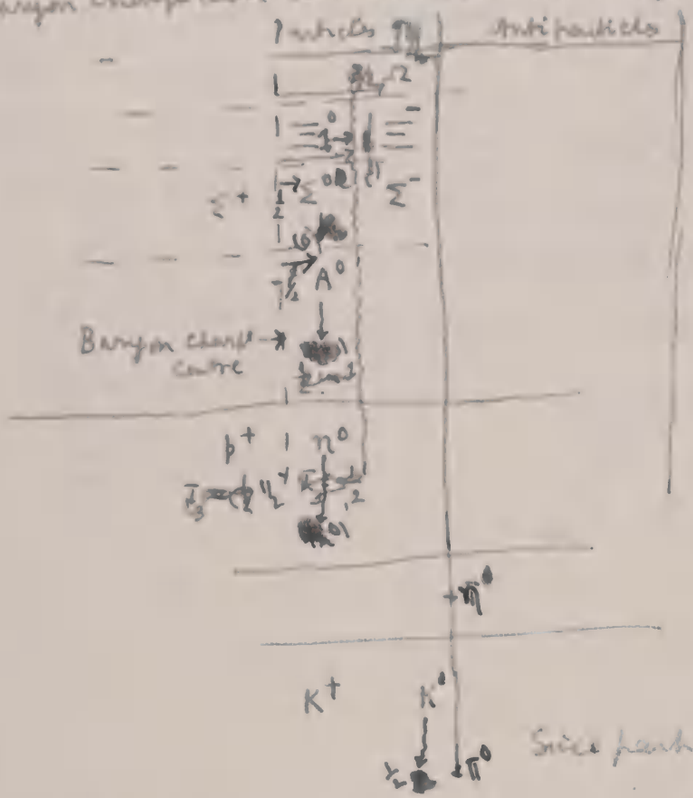
The nuclear forces do not distinguish between members of an isotopic spin multiplet, but when electromagnetic field is "turned on" differentially charges +, 0 or −, there is a separation measurable in terms of mass difference.

Isotopic spin labelling is only useful for baryons and mesons, and although values of I could be assigned to leptonmultiplets, it would not help describe processes in which they are involved

Just as justification for Mendeleef's table was that it worked, this labelling and prediction of new elements, so also the labelling by means of I works also works.

(2) Strangeness — Defined w.r.t. chart of elementary particles where, for eg, in the case of baryons, the particles are arranged columnwise + under +, − under −, neutral under neutral. The electric charge centres of the isotopic multiplets do not however occur in the same vertical line, and strangeness number

taking the baryon charge centre as the line halfway between p^+ and n^0, the strangeness number is defined as twice the displacement of the charge centres to the right in units of $\frac{1}{2}$ as measured from the baryon charge centre line. Thus for the charge centre of Λ^0 is at 0 (say at curved line constructed to n^0 thus making the nucleon charge centre at $\frac{1}{2}$), and hence strangeness = −1. Since the triplet isospin Σ is centred at 0 & has strangeness 0. In the case of the Ξ displacement of charge centre is 1 to the right ie $S = -2$; similarly for Ω^- displacement being $3/2$ to the right $S = -3$.



The same centre is used for the mesons, but the charge centre line got is chosen as that π^0. Thus for the doublet K^+, K^0, the displacement of the centre is $-1/2$ unit to the left hence strangeness $= +1$.

Since particles & anticharticles are mirrored in the chart, the antiparticles will have strangeness numbers opp. in sign to their particle counterparts — No use of notion of strangeness concept for leptons.

A relation between Charge Q, baryon number $\frac{b}{S}$ (ie $+1$ for baryon and -1 for anti-baryons), I_3 and strangeness number S are can be obtained in the form

$$Q = e\left(I_3 + b/2 + S/2\right) \quad (e = \text{unit electric charge})$$

Thus for p^+, $I_3 = +\frac{1}{2}$, $b = +1$, $Q = +e$, hence $S = 0$

" π^0, $I_3 = -\frac{1}{2}$, $b = +1$, $Q = 0$, " $S = 0$

" K^+meson, $I_3 = +\frac{1}{2}$, $b = 0$, $S = +1$ " $Q = +e$

" Ξ^0, $I_3 = +\frac{1}{2}$, $b = 1$, $Q = 0$, " $S = -1$., and so on.

The strangeness numbers were not introduced in this above ad hoc way. The original idea was in response to the problem of associated production of the new strange particles [vide Gellmann's article in Sc. Amer. only 5]. Foreg. the hyperons and K-mesons were never produced singly in high energy particle collisions, but always in associated pairs such as

Foreg: $\pi^- + p^+ \rightarrow \Lambda^0 + K^0$ (with $K^0 \rightarrow \pi^+ + \pi^-$). There is no apparent reason why one cannot not

have $\pi^- + p^+ \rightarrow \Lambda^0 + \pi^0$, but this is never seen in expts. If a hyperon is produced, a K-meson must also be generated. Also there is a further mystery viz. if a compound particle like the decays Λ^0, is going to decay into a proton and a π, why does it not do so almost as soon as it is formed? These two problems of associated production, and individual stability against immediate decay were those which earned the name of strangeness for the particles. Both these problems can be "explained" by insisting that the total strangeness involved in a fast nuclear reaction must remain constant. Since the hyperons have all $-ve$ strangenesses, they can be produced only in association with K-mesons having $+ve$ strangeness. Also once a hyperon or K-meson is produced and gets beyond the influence of the collision, it cannot decay without changing its strangeness, since π's decay products have strangeness zero. Thus in the fast nuclear processes, where total strangeness is conserved, strange particles can be produced only in pairs; once produced and separated from each other, they must wait for some weaker interaction to allow them to decay.

S is expressed in terms of another quantity called hypercharge Y, defined as equal to twice the average charge of a multiplet. For mesons $Y = 0$, for baryons $Y = S + b$ (since baryon centre line was taken to be that of the nucleon doublet at $+1/2$). Thus $Y = 0$ for Λ and Σ, -1 for Ξ and -2 for Ω.

(F) Decay times :- As shown by the chart, most of the particles are unstable & decay according to the exponential law $N = N_0 e^{-\lambda t}$ [N_0 = original no. present & λ is a proportionality const (not to be confused with wave length λ; another place)] - only particles stable against decay in vacuo are the proton, graviton, neutrinos (so far as we know), electron, positron & their antiparticles - Some decay schemes occur and some do not - for eg. none of the baryons decay into a combination not including a baryon; the proton cannot decay into a number of mesons & since no other law seems to forbid this, we hypothesise the law of Conservation of baryons. This is the "explanation" of the stability of the proton (makes no sense) ∥ ly

(E) The interactions. [(a) $F = G\frac{mM}{r^2}$, if the masses are measured in Kg. & distances in m., force & g in newtons]

(a) Gravity: Equivalence of inertial & gravitational mass the basic hypothesis of general relativity. If M = gravitational mass, m = inertial mass, for weight $\left.\begin{matrix} F_1 = G M_1 M_{earth}/r^2 \\ F_2 = G M_2 M_{earth}/r^2 \end{matrix}\right\} \therefore \frac{F_1}{F_2} = \frac{M_1}{M_2}$; for acc^n $\left\{\begin{matrix} F_1 = m_1 a_1 \\ F_2 = m_2 a_2 \end{matrix}\right.$

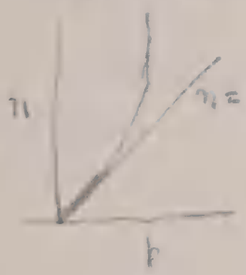
$\therefore \frac{a_1}{a_2} = \frac{F_1}{F_2}\cdot\frac{m_2}{m_1} = \frac{M_1}{M_2}\cdot\frac{m_2}{m_1}$ & if $\frac{M_1}{M_2} = \frac{m_1}{m_2}$, then $a_1 = a_2$ clarifying Galileo's exptl from leaning Tower of Pisa. Gravitational effect negligible in particle reactions & achieves significance only when the masses of interacting particles, and distances involved are vast. Just as photon is the agent of the e.m.f, the graviton is the agent of the gravitational field & having vel $\leq c$. Since strength of gravitational field is extremely weak.

the detection of individual gravitons is beyond our techniques now, and perhaps will be.

(b) Electromagnetism: Electricity & magnetism are both parts of the same phenomenon, a force far more complicated than gravity, not only are there + & − charges, but the interaction depends on the relative velocity of the charges. [(i) for eq. if $v = 0$, $F_{elect} = K \cdot \frac{q_1 q_2}{r^2}$ point charges in electrostatic case, there are limits of electric force between the charges; (ii) if $v = \text{const} = K$, the action can be partially described in terms of a magnetic field $B_{mag} = C \frac{i}{r}$ (i = current), (iii) if $\frac{dv}{dt} = a$ ie charge accelerated, radiant e-m energy leaves the scene - this given] - In Coulomb's law for electrostatic case, K depends on units chosen for the other variables. In the standard kg-m-kg-sec system, the unit of charge q is one coulomb. A current of one ampere is the passage of one coloumb per second. With F measured in newtons, $K = 9 \times 10^9$ n·m²/coul². In electromagnetism there is also a dependence on the medium; there being $\pm c$, if medium be an electric field, there are polarisation effects and there is no equivalent phenomenon in gravity — the coulomb is an enormous amount of isolated charge. F for a + & − 10^6 coloumb 1 metre away would be $= 9 \times 10^9$ newtons = wt of a million tons. But $+^{ve}$ or $-^{ve}$ coloumbs cannot be isolated in spite of this huge amount in a coulomb, a lowly flashlight battery can produce currents of one or two coulombs per sec, for an hour or more; the explanation is that $-^{ve}$ charges are flowing past equal amounts of $+^{ve}$ charge & neither being isolated in current flow does not require charge separation & any particular volume remains neutral. — Coulomb has most of the characteristics of Newton's gravit. formula, for eg, with are correct for isolated pt. sources or spheres, but in electrical case F may be repulsive or attractive since q may be $+^{ve}$ or $-^{ve}$. To compute strength of e-m interaction with nuclear binding, we have two protons 10^{-15} m. apart will be repelled by a force $F = 9 \times 10^9 \frac{(1.6 \times 10^{-19})^2}{(10^{-15})^2}$ ≈ 30 newtons. This is 10^{35} times as great as the gravitational attraction caused by this mass, but is only about 1% of the nuclear binding. For moving particles, there are both electric & magnetic fields, the latter being $\perp$ to the velocity & mag. of the charge and furthermore the strength is not directly towards the charge but is about $\frac{1}{3}$ of that from a direction of velocity. For cases of relativistic motion electromagnetic theory should be used; for eg. ordinary magnetism can be explained as an effect of the behaviour of moving electrons in the atoms or molecules. — Radiation from accelerated source charges; wave nature of radiation, radio 'photons'; wave and particle nature of radiation in terms of interference phenomena in terms of waves, and nature of β-rays (frequency of 3×10^{20} cycles/sec) - Table

Radio	Optical	X-ray
$\nu = 10^6$ cycles/sec	$\nu = 6 \times 10^{14}$	$\nu = 3 \times 10^{20}$
$E = 6 \times 10^{-34} \times 10^6$	$E = 3$ ev	$E = 1$ Mev
$= 6 \times 10^{-28}$ [illegible]		

(c) Strong nuclear force: There must be a special force associated with the nucleus; electrostatic repulsion of protons is large & must be overcome by some strong attractive force. Gravity is too weak & hence existence of a nuclear force. This force is independent of charge ie same between proton–proton, proton–neutron & neutron–neutron — No strength–distance or inverse square law for this force which must be short range forces. If two nucleons are touching they are strongly attracted, but when they are separated the force falls to zero, the range being $\sim 3 \times 10^{-13}$ m. which is about radius of nucleon. — Nuclear force depends on orientation, & the spin orientation requirements (governed by Pauli's exclusion principle for fermions) have great effect on stability of the nuclei. Symmetry in proton–neutron distⁿ leads to greater stability & the greater the asymmetry in p–n number, the faster will be the decay & more energetic the emitted β–particle — As nuclei get heavier they evidently are more stable if they have more neutrons than protons, as in diagram where the curved line shows contains the stable elements. The



heaviest stable nucleus is one of lead ${}_{82}Pb^{208}$ ie 82 protons and 126 neutrons. Bismuth (lead) is on the edge of stability with 83 instability (although $n > p$) because of the electrostatic repulsion of protons. The energy of repulsion goes up according to $Z(Z-1)$ (Z = no. of protons). Taking this & the rule of stability viz $n > p$ into consideration, it can be shown that the medium nuclei strike the most stable compromise; the lighter nuclei have too much surface and so not enough nuclear binding, while the heavy nuclei have too much electrostatic repulsion. [Refs (a), (b), (c) of nuclei of ${}_3Li^6$, ${}_{26}Fe^{56}$ and ${}_{92}U^{238}$ given] — Another picture of a nuclear force as arising from a field with the nucleus shown as a potential well — Picture of nuclear force on the analogy of electromagnetic interactions through the agency of photons, and Yukawa's proposal of 1935 that the agent for the nuclear force cannot be the photon but a particle of about 200 times m_e. At first the penetrating cosmic ray mesons were supposed to be these agents, viz. the mesons, but later on found to be the heavier mesons viz the Π mesons or pions (which are called the nuclear flux); however, theoretically the picture is not quite clear.

(d) Weak interaction:– Most of the particles listed in the chart have very short half lives (half life = $T_{1/2}$ = 0.69/λ, [(D)(S)]. The hyperons decay down into nucleons & mesons in times of about 10^{-10} sec; the pions decay into mesons in 10^{-8} sec, and mesons collapse to electrons in 10^{-6} sec. [1 nano-second = 10^{-9} sec] A suitable unit of nuclear time is the time taken by light to traverse the diameter of a nucleon ie 10^{-23} sec. From this point of view, the above times appear an eternity! — The strong interactions take place in times of about 10^{-23} sec. freely. During this interval pions & mesons (K–Mesons) are emitted and absorbed and so on their way, get some of the resulting particles, although energetically unstable, suffer no decay until a time 10^{13} greater has come & gone. The name is borrowed from β–decay, sometimes decay times of 1 second appear—

If only strong interactions are involved reaction should take place in nuclear times. If only electromagnetic force is involved, the time of reaction should be longer than this by a factor of 100. Hence if a reaction does not occur in these times, there must be a new conservation law or prohibition which forbids the decay, since the participants are both subject to nuclear or e.m. forces or both. But since eventually the decays do happen, there must be a fourth type of interaction which is not subject any rule of prohibition. The existence of such a force was proposed by Fermi in the early 1930's to explain β-decay, but we still know very little about it. Its relative strength is about 10^{-13} times the strong nuclear force & is called the weak interaction — Numerous & complicated examples of weak interactions quite dissimilar type, involving 4 fermions & 2 fermions & so on:

(1) $n \to p + e^- + \bar{\nu}_e$ with $t_{1/2}$ = 700 secs. & no change in strangeness (four-fermion involved)

(2) $\Lambda^0 \to \pi^- + p$ " " = 1.7×10^{-10} Sec & strangeness change +1.

[These mean that in a weak interaction S may or may not be conserved.]

"Eg/n" (1) & the neutron decay is the prototype of all β-decays. The nature of such an eqn is that the reaction can go in either direction so long as energy is conserved, and that any participant may be replaced on the opposite side by its anti-particle. For instance, such a prototype for e^+ decay would be

(1)' $p \to n + e^+ + \nu_e$, but an isolated p will not decay like this because it is lighter than n;

what participation of this 4-fermion interaction which has been observed is

(1)'' $\bar{\nu}_e + p \to n + e^+$

Another example of 4-fermion interaction is (2) muon decay given by $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$ — (3)

[ν_μ & ν_e different]

which has also been experimentally observed. In addition to (1), (1'') & (3) one also has the 4-fermion interaction (including a hyperon p) viz. $\mu^- + p \to n + \nu_\mu$ — (4) has also been observed.

For (1), (1''), (3) & (4) the strength is the same viz. that of the Universal Fermi Interaction.

Now consider (2); here only two fermions Λ^0 & p are involved & $\Lambda^0 \to \pi^- + p$ ($S=-1 \to S=0 \quad I=0$). No neutrino is involved & strangeness changes by +1. The unusual (2) can be transformed into two decays each involving 4 fermions, the first one being a virtual process (which must be considered because of Heisenberg's uncertainty Principle; explanation not clear & imprecise) as follows:-

$$\Lambda^0 \to \underbrace{(\bar{p} + n + p)}_{\text{virtual}} \to \pi^- + p \quad \times$$

[In this diagram, the virtual interaction is about as 10^{-24} sec in duration, but it is possible. Even though the V. nn lasts a long time, the S.N. nns take only 10^{-23} sec to act. No wonder]

Theory is lacking to calculate strengths of the 1st step & the 2nd step involving strong interactions. The virtual processes give correct results in e.m. interactions & in weak interactions give qualitative explanation of many of the observed phenomena — The question also arises whether there is an analogue to π for strong nuclear force & the photon for e.m. force in the case of weak interactions also. If there were a similar agent, it would also be a boson & would probably have a mass above 800 Mev. It is referred to as the Intermediate Charged vector boson & given the symbol W. Its half life of this decay would be about

or muon-neutrino would be less than 10^{-17} sec. It is unlikely that so far (1964) W has been detected. The data so far obtained indicate that if W exists, its mass must be > that of the nucleon. Perhaps in a few years this problem will be solved, only to reveal more subtle and complicated questions that we do not now realise exist — Not all particles are subject to all four types of force. For eg. The photons are agents of the e.m. force, but are not influenced by e-m fields. Neutrinos have no e-m interactions at all. The electron, muon, the neutrinos and photons do not seem to be subject to the strong nuclear forces. Indeed, the neutrinos are subject only to the weak interactions, and presumably to the far weaker gravitational forces. There are laws governing the behaviour of particles under the influence of these forces and not all laws apply to all forces.

[Then follow a list of 14 beautiful plates showing different types of interactions etc]

(F) The Conservation laws.

(a) Conservation of mass-energy :- All natural phenomena can be consistently described as satisfying this law of conservation of energy - Example of the chain of reactions (extremely complicated) involved in the acceleration of an electron in a Vande Graff machine, showing that as the complicated sequence proceeds step by step, the total mass-energy is conserved from beginning to end.

(b) Conservation of momentum :- valid for all interactions. For low velocities, momentum $= m.v$. In particle reactions with velocities close to c, $p = \frac{\sqrt{E^2 - (m_0c^2)^2}}{c}$. For photon or neutrino $m_0 = 0$ and $p = E/c$. Particles can have momentum without having rest mass at all; photons can indeed exert force, which is merely transfer of momentum. This is seen on the microscale in the Compton effect, the billiard ball collision of a photon with an electron.

Momentum conservation in x-direction $\rightarrow \frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + \frac{m_0 v}{\sqrt{1-v^2/c^2}}\cos\phi$

" y-direction $\rightarrow 0 = \frac{h\nu'}{c}\sin\theta - \frac{m_0 v}{\sqrt{1-v^2/c^2}}\sin\phi$

Energy conservation requires $h\nu = h\nu' + \frac{m_0c^2}{\sqrt{1-v^2/c^2}} - m_0c^2$

There are 3 eqns for 4 unknowns θ, ϕ, ν, ν'. Changing ν to c/λ (λ = wave length) and solving for θ, we get

$$\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\theta)$$ i.e Change of photon wave length is a simple function of its deflection angle, which is the Compton effect. Momentum conservation requires that since there was no momentum originally in the y-direction, after the reaction also the $\sum p_y$ must = 0 i.e. what goes up must = what comes down. In the x-direction the original momentum must be conserved. [There a printing mistake of proton for photon, on p. 76] ||x — Rest of this rel. expn for p for low velocities — An example of importance of this conservation law is that an x-ray of one MeV cannot, in free space, change into an electron pair, although all other rules would be satisfied. Momentum of energy $= p = \frac{E}{c} = \frac{1\,MeV}{c} = 5 \times 10^{-22}$ kg.m/sec. In terms of energy, this momentum is

is enough to turn the energy of the x-ray to the energy of the electrons plus some spare momentum, but this spare not enough to provide K.E. for the electron pair & hence the electrons/e have no momentum. Since the spare momentum cannot just disappear, the reaction does not take place. The reaction takes place only in the neighbourhood of a nucleus or electron which can share the spare momentum. [not quite clear how the extra K.E. is obtained in this case] // X.

(c) Conservation of angular momentum:- A.M. can be so defined that it is considered a strictly conserved quantity of an isolated system. The assignment of spin & orbital A.M. to the electron produces a theory successful in correlating all known expt'l. facts about the atom. In particle interactions we must insist that the total A.M. of the system remains constant. Each of the participating particles may have intrinsic spin even though they are at rest w.r.t. each other. A have no relative A.M. for eg. when a positron finally slows down in material it captures an electron; then the A.M involved is that due their spins lined up together or opposed leaving 1 or 0. The annihilation mass turns into e-m energy of photons each of which has spin 1. one photon alone cannot be produced because this would carry off linear momentum where there was originally none. The individual photons may be lined up or opposed yielding 2 or 0. only the latter is possible because of conservation of A.M. This is the most usual case & expts. have shown the resulting gamma rays polarised in the same direction. But there is one other possibility of three photons being emitted if e-p pair has aligned A.M.1; such a combination can conserve linear momentum and also contain three spins of one each to allow a total of 1. This 3-photon annihilation is also observed, but frequency $\frac{1}{1000}$ of the 2-photon process. — Rel A.M or spin is always quantised; spins may be $0, \frac{1}{2}, 1,$ & rel. ang. mom. must always be 0, 1, 2 or some higher multiple of $\hbar$ ie A.M is quantised just like charge — Rel A.M. of two particles makes itself apparent in scattering expts. for eg. 2 protons with spins aligned cannot exist in the same region because of the Pauli exclusion principle, and a head-on collision of two protons, with no A.M. due to their relative motion is forbidden. It is allowed however if the two spins are opposed. with protons bombarding neutrons, this prohibition does not exist.

(d.) Conservation of electric charge: All expt'l. evidence indicates that electric charge is a particular restricted type of quantised attribute, with particles having charges +1, 0, −1 units e. When charged particles are created out of energy, an equal quantity of the opposite charge always appears — Cosmologists have hypothesised that +ve charge may be slightly > the negative one & used it to explain the expansion of the Universe, but recent expts. have shown that the magnitudes of the electron & proton charge cannot differ by more than one part in 10^{20}, this ruling out the hypothesis of the expansion.

(k.) Conservation of baryons & leptons: Baryons are conserved and also leptons, but both

only separately. For eg, although all the conservation laws could be satisfied by a ~~reaction~~ reaction producing single baryons & leptons, such a reaction is never seen. Thus $\tau \not\to p^+ + e^-$. There are no such conservation laws for the mesons or photons. At present, there seem to be two separate kinds of leptons muon and electron, each with its own neutrinos and each with its separate conservation law. Hopefully a deeper understanding of the muon-electron problem may eliminate this redundancy [Vide Feynman's article on muon in Sc. Amer. July 1961 where this redundancy is described as a "riddle wrapped in a mystery inside an enigma"]

(e) Conservation laws of limited ~~jurisdiction~~ jurisdiction. - For an isolated system, energy-mass, momentum, A.M, electric charge, baryons and leptons are conserved regardless of which of the four types of forces is involved. There are some other quantities which are conserved in some types of reactions, and not in others. They are given below.

(f) Conservation of strangeness: Total strangeness of a system (Vide chart) remains the same so long as only strong nuclear or e-m forces are involved. For weak interactions, the strangeness number can change by no more than one unit. For eg. the cascade hyperon Ξ^- (the Ξ with strangeness -2 can decay to a nucleon through the weak interaction, but only by a two step process, thus

$$\Xi^- \;(S=-2) \longrightarrow \Lambda^0 \;(S=-1) + \pi^- \;(S=0) \qquad \Delta S = +1$$

$$\Lambda^0 \longrightarrow \pi^- \;(S=0) + p^+ \;(S=0) \qquad \Delta S = +1.$$

[For weak interaction S may or may not be conserved. See p. 160 of ref (——); it could be by more than one unit]

[The 2nd sentence on p. 82 viz "The strangeness no. can't go from -1 to 0, and this is forbidden" appears to be wrong] [It is right: he is talking of strong interaction only]

(g) Conservation of parity: - Physical events to be independent of the position or orientation of the observer and therefore the descriptions must be made similarly independent - It can be proved that if physical descriptions are independent of the origin of our coord. system, then momentum will be conserved. If description does not depend on rotation of coord. system, A.M is conserved. If the chosen setting of setting of clocks makes no differ to physical laws, then energy-mass is conserved. If our laws are independent of zero of electric potential, then electric charge is conserved - Another change of coordinates $(x, y, z) \to (-x, -y, -z)$ changing right hand to left hand i.e mirror reflection (unlike rotation through 180° which leaves a right handed system still right handed) is of interest & we can ask if there are events in nature which depend on the right handedness or left handedness of our descriptive system. Until 1956 it was a basic belief in physics there was no natural event with a preferred screw sense (i.e unlike to mirror reflection), but during the middle 1950's a paradox had arisen in the interpretation of certain phenomena of strange particles. This was the θ-τ paradox solved by Lee & Yang - Reaction involving the strong interactions & the e-m. interactions had been sufficiently studied so that there was

there was no question but that they were independent of handedness. But Lee & Yang soon realized that no explo. relativ to weak interactions had been made to test this point. They found that if weak interactions were dependent on screw-sense, it could explain the θ-τ paradox & would imply existence of this reaction depending on screw-sense — first expt of non-conservation of parity in β-decay confirming the ideas of Lee & Yang & its explanation — Expt of Wu & Ambler showing strongly non-conservation in Λ° decay, π mesons being preferentially on one side of the plane (thus defining "up", defining a screw sense & showing that nature can distinguish handedness — This asymmetry is nothing unnatural, for in distant galaxies with anti-matter this asymmetry may be reversed. The reflection of space coordinates seems to be tied in with the reflection of charge, turning particles into anti-particles. There appears to be symmetry only for the product of the two effects together — Mathematically one can talk of odd or even parity according as the wavefnctn changes or does not change sign under mirror reflection. Since it is found all physical processes conserve parity means ministry that there can be no process which produces a preferred space orientation. This conservation law was assumed to be as inviolate as that for charge or energy. Was we realize that only strong & e.m. interactions conserve parity, while the weak interaction seem to violate it every chance it gets - The θ-τ paradox. This was the bothersome behaviour of the K-mesons which for several years was called the τ-meson. Actually only 6% of charged K-mesons decay into three charged pions. These mesons, as many K's decay into just two pions, one charged & one neutral. These were called θ-mesons. By measurements it became more & more certain that the mass of the θ was the same as the mass of the τ. Furthermore everything about them seemed the same. The embarrassing feature of the θ-τ decay lay in the question of the parity of the parent K-meson. Experiment showed that the π-meson must have odd intrinsic parity. Question about is the intrinsic parity of the K-meson as it decays into pions? Since in the θ mode only 2 pions come out, necessarily in spp. wave and so necessarily with zero ang. mom. rel. to, the combination of the two odd-parity pions must be even. But in the τ decay, the intrinsic parity is odd, which led to τ having odd parity. The paradox was that one & the same particle appeared to have sometimes to have odd parity & sometimes even parity. The resolution of the paradox lay in the belief that parity need not be conserved in weak interactions

(h) Conservation of isotopic spin: Phenomena of "mirror" nuclei ie x protons & y neutrons → y protons & x neutrons, for many same interactions as long as spin arrangements are the same (original notion of nuclear interaction being independent of electric charge) — These phenomena explained by stating that in strong interactions isotopic spin is conserved. The law is violated by the electromagnetic and weak

In all strong & e-m interactions I_3 is conserved.

interactions. Examples:

(1)	p	+ p	→	p	+ p	+ π^0	
I	½	½		½	½	1	
I_3	+½	+½		+½	+½	0	

Re. conservation of I in this example, total I_3 and. H.S. & Total I can be 1 or 0, but I ≠ I_3 & hence I = 1

on the R.H.S, total $I_3 = 1$, but I can be 2, 1 or 0 & hence 0 is ruled out & conservation law is satisfied by taking $I = 1$ on R.H.S also. To show that I_3 is conserved in strong & e-m interactions, we note from $Q = e\left(I_3 + B/2 + S/2\right)$ that, since Q is conserved, and there is a change in I_3 only when there is a change in S but this change in S can happen only for weak interactions.

(2) $\pi^0 \longrightarrow \gamma + \gamma$

$I = 1$ no I

$I_3 = 0$.

Here I_3 is conserved because no weak interaction is involved. But I is not conserved as will be shown later since the decay occurs in a complicated way & I is not conserved for e-m interactions

(3)

	Λ^0	$\longrightarrow$	π^-	$+ p^+$
I	0		1	1/2
I_3	0		−1	+1/2

This is a weak interaction, and (i) I_3 is not conserved (it is 0 on left & $-1/2$ on right), (ii) π^- is emitted preferentially in one direction and parity is not conserved, (iii) charge is conserved which means that strangeness is not, & (iv) no combination of $I = 1$ & $\frac{1}{2}$ on R.H.S. can produce 0 & so I is not conserved. (most stupid reaction with no conservation except Q!)

(4) (π^-, p) and (π^+, p) collisions; types of reaction possible for the two are different. The π^+ collision is first limited to its just bouncing elastically from p i.e.

(i) $\pi^+ + p \longrightarrow \pi^+ + p$

I	1	1/2	1	1/2
I_3	+1	+1/2	+1	+1/2

(p. 92) This is a Breit-Wigner resonance with $I = 1$ for π vide chart

The π^- collision can be both elastic or charge-exchanging i.e.

(ii) $\pi^- + p \longrightarrow \pi^- + p$

I	1	1/2	1	1/2
I_3	−1	1/2	0	−1/2

or (ii)' $\pi^- + p \longrightarrow \pi^0 + n$

I	1	1/2	1	1/2
I_3	−1	1/2	0	−1/2

All the three reactions are strong and I or I_3 both I_3 and I must be conserved. In (i) with π^+ only one isotopic spin state is possible with $I = 3/2$ that $I = 3/2$ since $I_3 = 3/2$. In cases (ii) & (ii)' $I_3 = -1/2$ and so I can be either $3/2$ or $1/2$. Strong nuclear interactions depend only on the total I, the π^- results should be a combination of the π^+ results & the special contribution due to $I = 1/2$. The cross-section - kinetic energy curve shows this effect [See figs on left]

[The whole explanation is not clear. Does fig (i) correspond to (ii) or (ii)'. Also notion of resonance has been introduced here, although it is what is explained much later on] &

Later added: fig (ii) corresponds to both (ii) and (ii)']

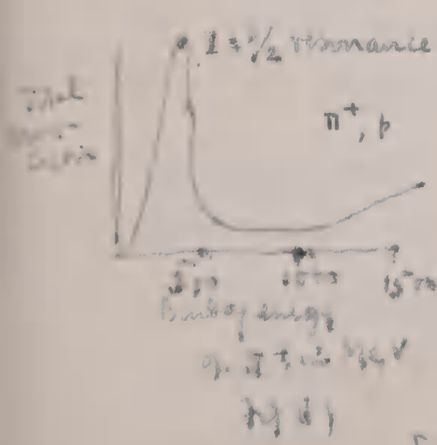


fig (i)

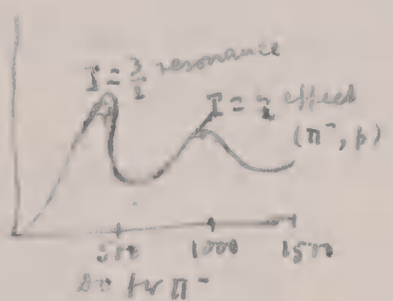


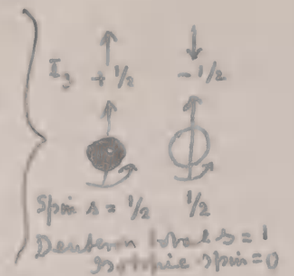
fig (ii)

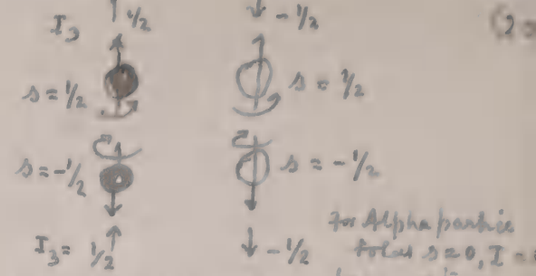
[Resonance for sound is sound coming back & reflected]

[Resonance means reaching an excited state and then coming back to original] !

(5) Creation of helium by deuteron fusion

	H^2	+ H^2	→ He^4	+ π^0
I	0	0	0	1
I_3	0	0	0	0





(2)

Unlike the four case examples cited above, this is an example of a reaction which does not occur, because it do so. Both the deuteron and the π-particle (He-nucleus of 2p, 2n) have I = 0 [statement here is I of deuteron or alpha particle = zero; should be ½ instead of 0?]. The plausibility of this can be seen by using $Q = e\left(I_3 + b/2 + S/2\right)$ · for H^2, Q = e, S = 0 and b = 2 and $I_3 = 0$. That I = 0 could be derived from the Pauli Principle for two fermions which here requires $I_3 = \pm\frac{1}{2}$ for p and n, leading to $I_3 = 0$ hence I = 0. for He^4, the arrangement shown above is the only one consistent with the Pauli Principle extended to include isotopic spin and leads to I = 0 (also from Q = 2e, b = 4 & S = 0 → $I_3 = 0$ → I = 0). We also notice incidentally that total spin s of H^2 & He^4 is 1 and 0 respectively as they are both bosons not subject to the Pauli rule. In reaction (5) above all conservation laws can be satisfied, but only that of isotopic spin is violated & this cannot happen in strong nuclear reactions. The process has been sought many times, but it could easily be seen if it occurred. No evidence for the reaction has been found.

(G) Roll call of the particles stable against rapid decay through strong nuclear forces.

The procedure hitherto adopted will be reversed, and the characteristic of each particle will be given in terms of the parameters that have been derived, starting from the bottom of the chart.

(a) Massless bosons

(1) The photon (γ) - Wave and particle models → $E = h\nu = hc/\lambda$; $p = E/c = h\nu/c = h/\lambda$

$E = \sqrt{(m_0c^2)^2 + c^2p^2}$; $p = \frac{\sqrt{E^2 - (m_0c^2)^2}}{c}$ which shows that a particle need not have rest mass to exert momentum - Even if in principle photon rest mass cannot be measured ∴ rest of photon = 0. — Spin of photon = 1 & hence a boson; is stable, no conservation laws forbidding creation & annihilation of photons — Photon can interact with particle only through their electric charge, usually involving electrons — Energy of photon of visible light is a couple of eV, which is just the energy needed in chemical reactions & so these photons are seen by our eyes, and produce photosynthesis in plants — If a photon delivers all its energy to an atomic system and causes electron emission, we have the "photoelectric effect" — The Compton effect clearly indicates the particle nature of the photon — When photon has energy 1·02 MeV, electron pair production can take place · The probability of this reaction increases with the energy which is responsible for explanation of electron-γ cascade showers in meteorites & also cascade showers in atmosphere starting with high energy photons from decay of π^0-mesons coming from the primary cosmic ray bombardment at the top of the atmosphere.

(2) The graviton - Zero mass, rest E, spin 2 - 'Gravity waves' not yet detected.

(b) The Leptons.

(1) neutrino associated with the electrons ($\nu_e, \bar{\nu}_e$) – Failure of the laws of conservation of energy, momentum and A.M. in β-decay (emission of e^+ or capture of e^- by nucleus) for eg: $C^{14} \rightarrow N^{14} + e^-$ where nuclear spins $S=0 \quad S=1 \quad S=\frac{1}{2}$ (S = spin) in (1933) of $_6C^{14}$ and $_7N^{14}$ are 0 and 1 – Pauli's hypothesis of each β-decay being accompanied by a neutral particle & called it 'neutrino' – Fermi's theory of β-decay using ν_e of small mass (Fermi called it 'neutrino') & explo demanding its mass ≥ 0 – ν_e is very penetrating & seldom interacts with anything. On going through the entire earth a ν_e of low energy would show only one chance in 10^{10} of causing a reaction – Detection of antineutrino using the enormous flux of these from a nuclear reactor in 1956 on the basic reaction $n \rightarrow p + e^- + \bar{\nu}_e$ & according to principle of lepton conservation, the inverse reaction $\bar{\nu}_e + p \rightarrow n + e^+$ also holding & leading to the detection (expt. arrangements also given) – Another expt. performed near a reactor further showing that ν_e and $\bar{\nu}_e$ are different – Rarity of interaction of the ν's with other particles is due to the fact that they are not subject either to the strong nuclear reactions or the e-m force, but only show up in weak interactions (although many weak interactions do not involve them). Since weak interactions always violate parity, Lee & Yang proposed a revision of the β-decay theory, assuming that the ν's are asymmetric w.r.t reflection of space coordinates. This coupled with the fact that ν's travel with vel. c showed that their spin must be oriented either along the line of sight or opposite to it. If parity were conserved, there would be equal probability, but expt. has shown that the spin of $\bar{\nu}_e$ is always along the direction of motion, while that of ν_e is opposite to its direction of motion. Such a theory, if $\bar{\nu}_e$ would advance like a right handed and the ν as a left handed thread; also mass of ν_e or $\bar{\nu}_e$ is assumed $= 0$.

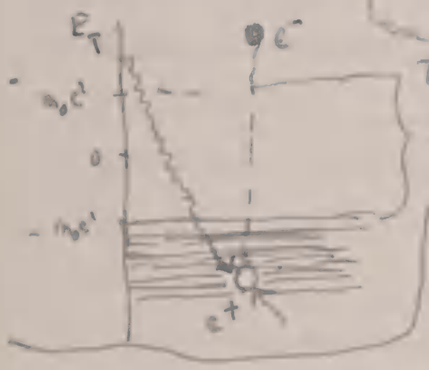
(2) Neutrino associated with the muon ($\nu_\mu, \bar{\nu}_\mu$) – $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$, the $\bar{\nu}_\mu$ being necessary for conservation of momentum. The subsequent decay of the muon must be $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$; also a small probability exists for a four-body decay viz. $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu + \gamma$; if ν_e & ν_μ were assumed identical, this reaction would lead to 5 possible reactions through bubbles (including without ms) which have never been seen [this part not quite clear] – Other actual expts. can be made to test the identity or otherwise of ν_μ & ν_e. In such expts the possible reactions to explain the results are

$$\nu_\mu + n \rightarrow p + \mu^- \quad \text{or} \quad \nu_\mu + n \rightarrow p + e^-$$

$$\bar{\nu}_\mu + p \rightarrow n + \mu^+ \quad \text{or} \quad \bar{\nu}_\mu + p \rightarrow n + e^+$$

& During an estimated exposure of 10^{14} neutrinos in a run taking 25 days of operation at the Brookhaven Laboratory Alternating Current Accelerator, 50 pictures were taken & none showed tracks which might be interpreted as due to electrons, but could be interpreted as those resulting in the L.H.S of ν_μ. It appears, therefore, that ν_μ cannot produce electron events & is therefore different from ν_e – The mystery of the existence of ν_μ & ν_e is almost certainly the same as the mystery of the existence of the muon, which in every way except its mass (and associated neutrinos) is the same as the electron. Whether or muon mass (i.e. mass = only 6% by S. Perlman) [illegible] expts with giant accelerators planned to resolve this.

(3) The electron (e^+, e^-). – mass = 9.1×10^{-31} kg = 0.51 MeV; charge = 1.6×10^{-9} coul ±; spin ½ ħ is a fermion subject to the Pauli Principle – not subject to strong nuclear force, but responds to the other three – no way for decay & still conserve electric charge & thus it is stable – Because of small mass attains rel. speeds at comparatively low energy for eg. 1000 eV – easily influenced by magnetic fields – δ-rays are electrons knocked forwards with energies of several KeV by a heavy ionising particle (specially doubly charged) goes through a cloud chamber – e^+ as stable as e^- but in our Universe it cannot last long; if e^+ & e^- come close with opp. spins, annihilation occurs with a half life of $\sim 10^{-8}$ sec. & two γ are emitted in opp. dirns to preserve momentum & since original A.M = 0, the photons are polarised so that their total spin is also 0 & this is observed – In about one case in a thousand annihilation occurs with spins of e^+ & e^- aligned; the life time for this is 1000 times greater & with A.M = 1 & 3 photons are emitted all in one plane & polarised so as to produce spin one in total. – Mutually e^+ was the first anti-particle detected – Dirac's theory of holes identified theoretically; if e^- in −ve energy state is given sufficient energy, electron is formed & the hole left is the positron – Disintegration of e^+ by emission, and the interpretation of Plate 41.



Total photon energy req'd = $2m_0c^2 + E_{kin}(e^-) + E_{kin}(e^+)$

forbidden region

(4) The meson (μ^+, μ^-) – In every way $\mu^\pm$ are merely heavy $e^\pm$; no nuclear interactions; their e-m interactions are the same as $e^\pm$ in all details, only mass is 207 times as great as $e^\pm$. $\mu^\pm$ are usually formed as decay products of the π− and K−mesons. There is low probability that muon pairs will be produced by γ-rays. The muon has a life time 100 times as long as that of π, which does not travel far due to its short life time after being produced by high-energy collisions of primary cosmic ray protons at the top of the atmosphere. Even when travelling with speed c, half of the muons should decay in a distance of 450 m. it no more than 1/50 of the distance from their origin to the earth. The relativistic dilatation of time explains this anomaly. We observe that their decay times are longer than if they were at rest; in their frame of reference, the distance between origin & earth has shrunk by the same factor – Spin of $\mu^\pm$ is ½ ħ is same as that of $e^\pm$. In all but light nuclei μ^- will be captured and absorbed into the nucleus in times shorter than the microsecond needed for decay; in light nuclei (lighter than carbon) decay takes place before absorption. The μ^+ cannot be captured into an atomic system & so are always seen to decay – The nature of π-μ-e decay will be detailed –

mass of μ = 207 × 0.51 MeV ≈ 106 MeV

(C) The mesons:

(1) The pi-meson (π^+, π^-, π^0): Short-range nuclear force is created by interchange of mesons (π is a 1st approx? to the nuclear interaction agent proposed by Yukawa in 1935) & since such interaction agents must be capable of being produced one at a time, there can be no conservation law about their number. They have spin zero, and are bosons – Cosmic-ray particle found in 1937 by Anderson et al had same mass predicted by Yukawa, but their existence at sea level, having come through whole atmosphere, indicated that their interaction with matter was slight & hence could not be the nuclear glue & this was clear in 1941. Next year it was possible to produce π-mesons at the Berkeley synchro-cyclotron — mass of π ≈ 140 eV – π^0 is its own antiparticle (Spin 0) – Produced if sufficient surplus energy be given to a nucleon system, also through Coulomb [illegible]

energy photons, and also produced in weak decays of the hyperons ie they are subject to all four interactions — Decay of $\pi^\pm$ must take place through the weak interaction since the nuclear forces do not affect ~~mesons~~ particles that are lighter than π & no e-m route is possible. For eg. $\pi^- \to e^- + \gamma$ ($S: 0 \to 0 + 0$; $\frac{1}{2} \to \frac{1}{2} + 1$) is not possible since it would mean creation of a single lepton and also spin is not conserved. The actual decay route (in 2.8×10^{-8} sec) is $\pi^- \to \mu^- + \bar{\nu}_\mu$; also $\pi^+ \to \mu^+ + \nu_\mu$. In about one time in 10,000, the alternative mode with an electron viz $\pi^- \to e^- + \bar{\nu}_e$ & $\pi^+ \to e^+ + \nu_e$ is seen. The weak interaction is always observed between pairs of fermions, but here the primary π is a boson. However, using the uncertainty principle a virtual state can be invoked viz for eg: $\pi^+ \xrightarrow[\text{stray}]{} (p^+ + \bar{n}) \xrightarrow[\text{virtual}]{} \mu^+ + \nu_\mu$ — The π^0 has a much more transient existence. An electromagnetic decay is open to it, because there is no electric charge to be conserved; this $\pi^0 \to \gamma + \gamma$, half-life being 0.7×10^{-16} sec [vide (H)]. This again must be subject to a virtual process since π^0 should not be subject to e-m field; in fact most likely virtual process is $\pi^0 \xrightarrow[\text{stray}]{} (\eta + \bar{\eta}) \xrightarrow[\text{virtual}]{} \gamma + \gamma$. An alternative process which occurs about once in 80 times is $\pi^0 \to \gamma + e^+ + e^-$, and the electrons in this case are called a Dalitz pair; in every 29 000 decays two pairs of electrons will be produced without any γ— There is a slight mass difference between charged pi's & π^0, π^0 being lighter by about 4 MeV. [Found from $\pi^- + p \to n + \pi^0$, & $\Delta E = (m_\pi - m_{\pi^0}) - (m_n - m_p) \approx 4$ MeV] [vide also Sachs (H)].

(at rest) [for $\pi^\pm$, $I_3 = \pm 1/2$; for π^0, $I_3 = 0$, so that from $Q = e(I_3 + 1/2 + S/2)$, $S = 0$ for $\pi^\pm$ & π^0] while $I = 1$ from chart]

(2) The K-meson (K^+, K^-, K^0, $\bar{K}^0$) — Produced by the strong nuclear force when particles interact at very short range ~~and~~ there is a great variety of decay modes of the K-mesons — Earlier confusion re. these decay modes referred to in (F) as the $\tau - \theta$ paradox, resolved by the discovery of non-conservation of parity in weak interactions [viz $\tau \to \pi^+ + \pi^- + \pi^+$, $\theta \to \pi^+ + \pi^0$) showing that both τ^+ & θ^+ are merely K^+.

— The ~~production~~ associated production of K mesons, as remarked earlier, was explained by conservation of strangeness (the baryons having -1 strangeness had be associated with K^+'s having $+1$ strangeness, in production — Spin of K mesons is zero as can be seen from their decay products (for eg. τ & θ decays); they can also decay into lepton pairs with spins which cancel. The decay times of ~~all~~ are 10^{-10} to 10^{-8} secs. are comparable to measure in spite of time dilatation ~~effect~~; Decays can also be through weak interactions since nuclear & e-m ones are ruled out by conservation of strangeness —

[for $K^\pm$, $I_3 = \pm 1/2$, for K^0, $I_3 = -1/2$, for $\bar{K}_0$, $I_3 = +1/2$ & so for $K^\pm$, $S = \pm 1$, & for K^0, $\bar{K}^0$, $S = \pm 1$ resp/ly, $I = 1/2$ for all K's]

— The fact that K^0 is not the same as $\bar{K}^0$ (unlike η^0 & π^0) produces behaviour unlike that of any other particles. [K^0 and $\bar{K}^0$ are particle & antiparticle although spin & charge = 0 for both, because strangeness is different] First of all it makes K a charge-doublet state with $I = 1/2$. If sufficient energy is available during production processes, K^0 or $\bar{K}^0$ can be produced to create the proper balance of S. During weak interaction decay, S is not conserved necessarily conserved, [In the book it is said 'not conserved', but about $n \to p + e^- + \bar{\nu}_e$ on p. 53] for eg: $K^0 \to \pi^+ + \pi^-$ or $K^0 \to \pi^0 + \pi^0$, and for $\bar{K}^0$, $\bar{K}^0 \to \pi^- + \pi^+$ or $\bar{K}_0 \to \pi^0 + \pi^0$. Thus during decay K^0 & $\bar{K}^0$ appear identical, while during creation, they are separate because of $S = +1$ or -1 — The

explanation of this anomaly by Pais & Gellmann that the decaying K^0 or $\bar{K}^0$ could be described as existing in a state with equal probability of being K^0 with $S = +1$, and $\bar{K}^0$ with $S = -1$, and that the decay states are a combination of the creation states and vice versa. They also use the quantum mechanical description of these probabilities being in phase or out of phase (interference effects: constructive or destructive) leading respectively to the emission of real π^+ & π^- or no real emission of the pi's — These decay states for constructive interference or otherwise are given different symbols K_1^0 (giving π^+ & π^-) and K_2^0. The decay time for K_1^0 is relatively fast $\cong 0.7 \times 10^{-10}$ sec. Since this decay is forbidden to the K_2^0, it has to wait for complicated routes to occur. These three-body decays involve π's or combinations of pions & leptons, the half life being about 4×10^{-8} sec. — This strange resonance effect leads to behaviour of different sorts. Once either K^0 or $\bar{K}^0$ is created, there is a 50% probability that it will decay with a half life of 0.7×10^{-10} sec. A beam of these will rapidly turn into a beam of K_2^0, as the K_1^0 die out with the two-pi decay. Further downstream, later in time, an almost pure K_2^0 beam will produce 3-body decays. Yet another strange effect can occur. Neither K_1^0 or K_2^0 has ["have" to be corrected] a definite strangeness assigned to it, but both are combination states that can be described as half K^0 & half $\bar{K}^0$. The K_2^0 beam can therefore produce effects in nuclear collisions which require either + or − S. The remarkable feature of this behaviour has been experimentally confirmed. If K^0's are produced in association with Λ^0 (S a− they have +1 strangeness. In subsequent nuclear collisions they therefore cannot produce Λ^0, which would require −1 S. However, if a beam of K^0 is allowed to travel some distance, the decays mode K_1^0 dies out, leaving mostly K_2^0. Each of these (3) is composed equally of K^0 and $\bar{K}^0$. Now Λ^0 can be created because $\bar{K}^0$ has −1 S. (Most baffling Kaons!)

(3) The eta zero (η^0) — This like the π^0 can decay through an e-m. interaction into π^+, π^- and π^0, this decay time being $< 10^{-16}$ secs. Most of the particles described (discovered) in this type of multi pion decays are the so-called resonance particles, unstable even against strong nuclear decay [Vide (H)] (ie unstable against rapid decay through strong nuclear forces) ($< 10^{-16}$ secs. time for decay is very short like that of π^0). The η^0 is not strongly coupled to its pion decay products, although there does not appear to be any theoretical reason against this. Nearly conservation law which would allow an e-m decay while forbidding a strong nuclear decay is that of I. Since η^0 has $I = 0$, and the pions have $I = 1$ with $I_3 = +1$ for π^+, $= 0$ for π^0 & $= -1$ for π^-, the π's can arrange themselves to have an $I = 0$, but the experimental evidence is that they do not.

(d) The baryons:

(1) The nucleons (p^+, n, $\bar{n}$, $\bar{p}^-$) — p & n are isotopic doublets with $I_3 = \pm \frac{1}{2}$ respectively. The e-m difference between them is also responsible for their mass difference. In the nuclei of

particle — Problem re. magnetic moment of neutron viz an uncharged spinning particle cannot form a magnet - ; explanation perhaps lies in internal structure of currents for eg. some $+^{ve}$ and some $-^{ve}$ regions. Also perhaps, the region at small radius might be considered to be the source of K-meson; and the outer region the source of pions — No such model so far proposed gives results in agreement with expt, also vide Segrè (M) — No conservation law forbids decay of p^+ into lighter particles, but it is stable and this stability is explained by university the law of conservation of baryons ($b = \pm 1$ & total no. conserved) — n is heavier than p^+ & will decay into p^+ & e^- with a surplus energy of 1·3 MeV & half life about 700 sec, and to observe this free decay (in contrast to the reactions inside the nucleus of exchange of p^+ & n through π-mesons), special arrangements are necessary — Earlier theories about nucleus being made of p^+ and e^-'s; untenability of electron being inside a nucleus [vide D (1), p. 7] due to uncertainty principle; also untenability in view of total spin for eg. if $_7N^{14}$ were composed of 14 protons (to account for mass) & 7 electrons (to yield a net charge of +7) its spin would be half-integral, while the measured spin is 1. This is reasonable in terms of a nucleus consisting of 14 fermions, 7 protons & 7 neutrons — [3rd term on p. 110 mentions "Bothe and Becker"; is this Correct? Should it not be Bethe & Backer ?] ✗ ? — Another earlier attempt to explain neutron effect by B & B by assuming very energetic γ-rays viz. the explanation of the penetrating radiation produced when α-particles from Polonium bombarded beryllium. Joliot & Curie discovered that the radiation would knock protons out of paraffin with an energy of over 5 MeV. To do this a γ-ray would have to have an energy of 50 MeV [for a γ-ray frequency is 3×10^{20} giving by $E = h\nu$ = 1 MeV] which is not possible. So Chadwick suggested the neutron in 1932 as combining the penetrating radiation, giving a reasonable K.E of 5 MeV to n; Many other expts showing existence of such neutral particles & disproving the notion of electrons being inside the nucleus — Production and detection of high energy neutrons (about 200 MeV) only by detection indirect methods like secondary detection of collision products — Antiprotons discovered in 1955 at the Bevatron in Berkeley by the reaction:

$$p + p \rightarrow p + p + p + \bar{p}$$
$$b: \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad -1$$

& details of expt. detection — creation energy for a nucleon nucleon pair is 1876 MeV, and although this bombarding energy is within the range of the 3 BeV Cosmotron, the bombarding energy must be much higher because of the Conservation of momentum. In the above reaction, the most efficient way to distribute the bombarding momentum is for each of the four resulting protons to share it equally. Thus $p_{\text{bombarding}} = \frac{4}{1} p_{\text{original}}$. If each of the final protons has the same energy, say E_1, then the original K.E of the bombarding proton must have been $E_0 = 4E_1 + 2m_0c^2$. The relation between K.E, total E,

and momentum is $E_{kin} = \sqrt{(m_0c^2)^2 + c^2p^2} - m_0c^2$. Hence $cp = \sqrt{(E_K + m_0c^2)^2 - (m_0c^2)^2}$

using $c^2p_1^2 = \frac{1}{16}(c^2p_0^2)$, we have $(E_1 + m_0c^2)^2 - (m_0c^2)^2 = \frac{1}{16}\{(E_0 + m_0c^2)^2 - (m_0c^2)^2\}$

$= \frac{1}{16}\{(4E_1 + 3m_0c^2)^2 - (m_0c^2)^2\}$, using $E_0 = 4E_1 + 2m_0$

Solving for E_1, we get $E_1 = m_0c^2$, and minimal K.E $= 4m_0c^2 + 2m_0c^2 = 6m_0c^2 = 5630$ Mev

(2) The hyperons $(\Lambda^0;\ \Sigma^+, \Sigma^0, \Sigma^-;\ \Xi^0, \Xi^-;\ \Omega^-)$ – from their decays, they appear to be composite of nucleons & mesons. Latest theoretical efforts have viewed all these particles as states of the same basic object – All hyperons have -1 or -2 S; originally they were seen in association with K-mesons of $+1$ or S; but with available energies large enough to create antibaryons, the strangeness and baryon conservation laws can be satisfied by creation of hyperon pairs; Eg of an $\bar{p}^- - p^+$ collision producing a $\Xi^- - p^+$

– Discovery of these new particles is a result of joint effort; most of them were theoretically postulated in 1958 by Gell-man & independently by Nishijima – In terms of I there is a singlet (Λ); triplet (Σ), doublet (Ξ or cascade), and singlet (Ω^-) ~~—~~ Because of their S, they must decay through the weak interaction with one exception viz $\Sigma^0 \to \Lambda^0 + \gamma$ which has an e-m decay mode available to it/ since both Σ^0 & Λ^0 have $S = -1$, the process is allowed.

– The Ξ has $S = -2$ & it decays into a nucleon by a 2-step process & was often referred to as the "cascade particle"; now the particle deserving the cascade title is Ω^- since its production process is $K^- + p \to \underset{-3}{\Omega^-} + \underset{+1}{K^+} + \underset{+1}{\bar{K}^0}$; the Ω^- decays through the $\underset{S=-1}{}$ weak interaction either to a Ξ or a combination of Λ^0 and K^-.

(H) The Resonances and Short-lived Particles

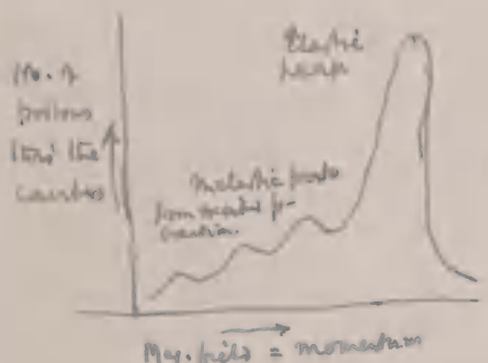
(1) Detection methods for very short-lived particles – question of existence or otherwise of short-lived particles can be partly answered by studying problems of detecting certain types of reactions – Examples (i) $\Lambda^0 \to \pi^- + p^-$ by strong nuclear force might take place in 10^{-23} sec, but it is forbidden since S is not conserved $(-1 \neq 0 + 0)$, but it can take place through a weak interaction in time $\sim 2\times10^{-10}$ sec, which is ~~always~~ adequate to allow detection of the process.

(ii) $\underset{I=1}{\pi^0} \to \underset{0}{\gamma} + \underset{0}{\gamma}$ ($t_{1/2} = 2.1\times10^{-16}$ sec) & $\underset{I=1}{\Sigma^0} \to \underset{0}{\Lambda^0} + \underset{0}{\gamma}$ ($t_{1/2} << 10^{-13}$ sec). In both cases only conserved is I is violated but this law is permissible for e-m interactions like (ii). To explain how the e-m force can bind electrically neutral particles to produce photons, following double sequences may be hypothesized viz $\pi^0 \to \bar{p} + p \to \gamma + \gamma$ & $\Sigma^0 \to \bar{p} + p + \Lambda^0 \to \Lambda^0 + \gamma$

of virtual states (governed by the uncertainty Principle) explains the short life times. Actually decay thro' e-m processes should take about 10^{-20} sec. but this is slowed down by the virtual processes, but by it cannot be calculated by how much the slowing down takes place, since there is so far no complete theory for the strong interactions coupling the mesons and baryons. In spite of the very short mean life times Σ^0 & Π^0 are easily detected by a special analysis. (iii) detection of π^0 by measuring the decay gammas or electron pairs: $\pi^0 \to \gamma + \gamma$; $\pi^0 \to \gamma + e^+ + e^-$ (1% of the time (Dalitz pairs). Analysis of many such events gives statistical evidence for the very short half life of 0.7×10^{-16} sec. (iv) Actual way of discovery of π^0 was through ~~the reaction~~ a different reaction viz $\pi^- + p^+ \to n + \pi^0$ (with $\pi^0 \to 2\gamma$). There is also a reaction $\pi^- + p^+ \to n + \gamma$. These can be distinguished (ie the two types of γ's) by the different distributions of initial energy between the decay products. Also one has the relation $m_0 c^2 = 2\sqrt{E_1 E_2}$, where E_1 and E_2 are the minimum and max. gamma energies (m_0 = mass of π^0); energy distributions of photons for the two cases of $\pi^- - p$ capture can also be shown graphically. There is a third 3-body decay in addition to the above two viz $\pi^- + p^+ \to n + \gamma + \gamma$ & energy distribution is also different for this. This difference in the division of energy among the final products is the key to use in searching for other short-lived particles - [& more irrelevant material] ✕ - The shorter the lifetime of the intermediate process, the more uncertain the energies of distributions.

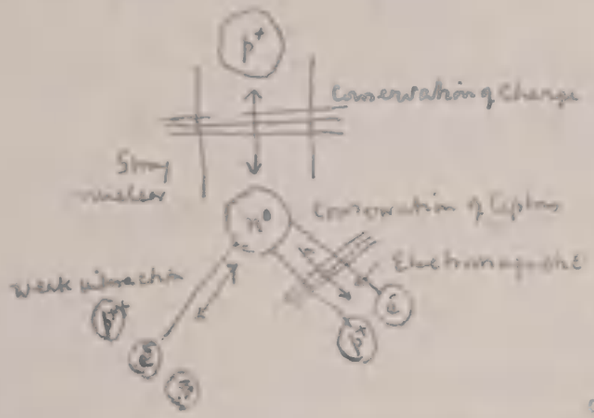
(2) Discovery of the particles unstable against strong nuclear decay: It is not clear whether such intermediate events should be called particles. It has been apparent for many years that the nucleons are complex particles; for some purposes they can be described in terms of a core surrounded by clouds of mesons. Such a structure might have temporarily excited states (N^* of Pais) known as the atom has excited states with the electron being further away & coming to original position with emission of a photon. Similarly, evidence (from 1950's) has been accumulated to show that the pion clouds of the nucleon might be arranged in abnormal ways in a collision, and bubble-chamber photographs of the envision of a meson - the most straightforward way of seeing these excited states is by bombarding the nucleon and observing the recoils. In $p-p$ scattering, we should have a particularly simple case of 2-body collision; description of a proton isobar (having same mass number) experiments; the results are that, although there is a peak in the curve (momentum-energy against no. of p^+'s through the counter) at the particular momentum corresponding to the elastic scattering, there are also protons which arrived with smaller momentum-energy. In fact, there appear to be several groups of these at preferred momenta. The position is analogous to what was observed in the expts of Franck & Hertz with electrons (40 years ago) as bombarding particles and the atoms as the target. The atom, in that expts, could accept energy only in

in discrete amounts which lift the electronic structure to quantised levels. From the separation of these states of excitation of the atom, its structure could be calculated. We come to the same conclusion in latter days about the proton. The curve on the left mentioned in connection with the proton isobar expt would be somewhat like the Frauenhofer - Hofstadter expt. curves on the energy of the scattered electrons. Thus the proton must have a structure which can be arranged in definite excited states. Each of these can be assigned, experimentally, a definite set of identification parameters, mass, charge, spin, strangeness etc. for eg. what appears to be the first excited state of the nucleon has mass 1238 MeV [MeV = 10^6 eV and not to Bev = 10^9 eV. This is called in European GeV], charge +2, +1, or, or −1; spin $\frac{3}{2}\hbar$; $I = \frac{3}{2}$ and $S = 0$. Whether we or not we call these configurations particles is all a matter of what sort of expt. we are doing with them. They do indeed have a brief independent existence (long enough to allow the excited proton to travel beyond the influence of the other proton, creating for a short time a 2-body particle reaction with simple A.M. relationships), but unlike the π^0, they are not even stable against decay through the strong nuclear interaction - Since we have claimed that in one sense all baryons may be parts of the same family, we should expect that all the heavy particles may have excited states. Such states have been found for eg. The reaction $\pi^- + p^+ \rightarrow \Lambda^0 + K^0 + \pi^0$ could have happened like this: $\pi^- + p^+ \rightarrow \Lambda^{0*} + K^0$; $\Lambda^{0*} \rightarrow \Lambda^0 + \pi^0$; similarly $\pi^- + n \rightarrow \Lambda^0 + K^0 + \pi^-$ could have happened like $\pi^- + n \rightarrow \Lambda^{0*} + K^0$; $\Lambda^{0*} \rightarrow \Lambda^0 + \pi^-$. An appeal to division of energy analysis enables the calculation of the mass of the excited state Λ^{0*} - There has been no success in explaining nuclear structure in terms of a core surrounded by clouds of π and K mesons. By 1960 theorists were calling for other types of mesons - Analysis of multiple pion production like $\pi^+ + p^+ \rightarrow p^+ + \pi^+ + \pi^+ + \pi^- + \pi^0$ or $\pi^+ + p^+ \rightarrow p^+ + \eta^0 + \pi^+$; $\eta^0 \rightarrow \pi^0 + \pi^+ + \pi^+$ showing that each positive triplet (or each doublet) might have come from the decay a single hypothetical particle (using energy & momentum analysis) - Many such combinations of multiple π production were analysed in 1961, and a large number of short-lived particles have now been identified.



(3) Significance of the short lived particles - In the case of excited states of atoms, weakness of the e-m interaction produces in the transient modes only a small correction in the energy level calculations. We have also, as a result of the weak interaction, considered each excited state of the atom as a separate atom, since the parameters of the ground state can be calculated without regard to those of the excited states - But whether or not the neutron can be considered a fundamental particle in its own right raises more complicated issues. It is not even stable by itself, but decays through the weak interaction:

Nevertheless during its existence it has all the attributes of a particle stable against fast decay through nuclear interaction. In view of the approach to excited states of the atom, we could perhaps be justified in claiming that all the baryons including the neutron are merely excited states of the basic nucleon. Also, there appears to be no evidence for the arbitrary separation of particles which decay through the weak or em interactions, and those which we have thought as resonances because they decay through the strong nuclear interaction — Question also arises whether the interactions are forces or processes. [At bottom of p.124 says " we have been speaking of interactions as if they were processes rather than forces", and first sentence in last paragraph on p.125, says " looked at in this way, the interactions lose their significance as forces". The first sentence should be have 'processes' & 'forces' interchanged] X Taking the case of the neutron which is responsive to all four interactions, and neglecting gravity, and also final states of higher mass ($\pi^- + p \rightleftarrows n + \gamma$), we can indicate the interactions as routes (with width of bands corresp. to strength of interactions) as follows in the figure.



looked at in this way, the interactions lose their significance as forces. The weak interaction need not be considered a repulsive force, but simply another route by which the baryon particles can transform into each other. All the interactions are two-way streets. The case where states of higher mass are considered can also be treated as route phenomena; one of the communicating states being $\pi^- + p^+ \rightarrow n + \pi^0$ (→ 2γ) — Another question (with which π^0 is related) is whether any of the particles responsive to the strong nuclear interactions have identities independent of the other particle combinations with which they can communicate. Perhaps the baryons and mesons are all creatures of each other and of the strong nuclear interactions. This model is meaningful in the case of strong interactions, but would not be useful in describing systems bound by the weaker interactions. The ground state of an atom can be described independently of the excited states. Apparently this is not the case with particles responsive to the nuclear interaction. There is no reason for considering one of the baryons as the ground state and the others as excited ones, since no one of them can be described in terms of the others. In this view of things, the proton is stable because, for some reason we do not yet understand, there is no baryon with a lower mass into which it can decay.

The interplay among baryons and mesons [hadrons]; the book does not contain this word, perhaps ~~discarded~~ used after the book was written] X? links together the existence of particles which can change into each other without violating any of the laws of conservation for strong nuclear interactions. This ~~suggestion~~ suggests a new classification scheme based on groups whose members have the same Y, Y and I. The resonances, or particles unstable against strong nuclear decay, are members in good standing of such groups. This organising scheme is due to Chew, Gellmann & Rosenfeld and was presented in Sc. Amer. Feb. 1964 issue. They propose a new naming system to avoid the confusion in the literature created by the discovery of so many new particles. The old name for the lowest

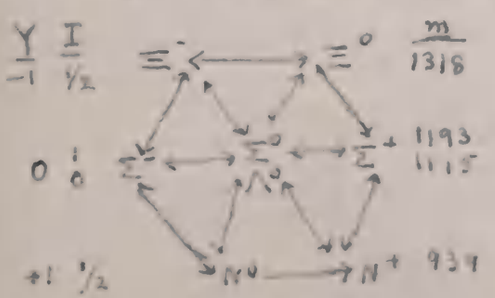
lying member of a family with particular b, Y and I, is used for the whole family. The mass, spin and parity of the member is then ~~for~~ summarized in parentheses. Thus e.g. the nucleon has $b=1$, $Y=+1$, $I=\frac{1}{2}$ and it is written as $N(939, \frac{1}{2}^+)$ ie mass 939 eV, spin $\frac{1}{2}$ and + hariby. [This classification is clear for the two tables (a) & (b) for baryons and mesons on p. 128, where the Ω ($Y=-2, I=0$), Ξ ($Y=-1, I=\frac{1}{2}$), Σ ($Y=0, I=1$), Λ ($Y=0, I=0$), N (nucleon) ($Y=+1, I=\frac{1}{2}$) and Δ (resonance of N) ($Y=+1, I=\frac{3}{2}$) and associated particles are given in five columns, with the rows corresponding to mass from 900 MeV to 2000 (in words 900, 1000, etc) ie 12 mass rows in (a) ~~situated~~ with $b=1$ for all baryons. Similarly in (b) four columns for mesons ($b=0$) viz. Antikappa ($Y=-1, I=\frac{1}{2}$), π ($Y=0, I=1$), η ($Y=0, I=0$) & Kappa(K)? ($Y=+1, I=\frac{1}{2}$) (the anti-kappa are ~~and~~ K^+ & K^-), and 13 rows from 0 MeV to 1300 MeV. But the classification Fig 69 is not at all clear. (should refer to Sc. Amr. Feb 64 article]. In fig 69, the notation ⬡ meaning a particle injitha with it is stable against nuclear decay is used, also ⬡ injitha a meson. Thus one has ⬡ nucleon; also generic type, ~~the~~ mentioned above, like $\Delta(1238, \frac{3}{2}^+)$ is used to denote that energy (mass) is 1238 MeV, spin $= \frac{3}{2}$ & parity $= +$. In fig. 69 there are 20 rows for MeV from 0 to 2000, 4 cols for particles with labels +2, +1, 0, -1, and 4 cols. for antiparticles with labels +1, 0, -1, -2 [It is not clear what these labels denote & it is not explained in the book either]. and there is another last column with 23 entries of all sorts with generic parameters given. In fig 70 (a) we have under Ω only one entry ⬡ (1676, $\frac{3}{2}^+$) ie the stable Ω^-_1; under Ξ (Xi), ~~two~~ entries $\Xi(1530, \frac{3}{2}^+)$ and ⬡ Ξ (1318, $\frac{3}{2}$), ~~the former being a resonance by the Kinkpachia rule, parity is shown, and the latter is the stable Xi particle of parity ~~undetermined~~ ~~since no~~ of~~~

π (782, 1⁻) and (π) (548, 0, −); under Kappa, two entries viz K (888, 1⁻) and (K) (496, 0⁻) — out of the resonance quadruplets of the nucleon N, two are known Δ (1238, 3/2 +) (ie the so called 3/2, 3/2 resonance of the nucleon viz both spin & isotopic spin 3/2), and Δ (1920, 7/2).

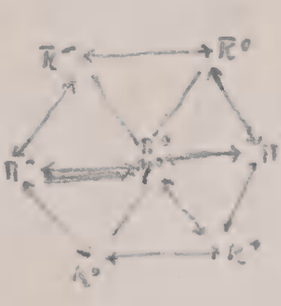
Suppose we assume that any particle is made up of all the other particle combinations with which it can communicate through the strong nuclear interaction (not involving leptons or photons). In a complete theory we should be able to show why these interacting systems appear with the particular properties we observe and no others. The mass of the nucleon, for instance, must have the value it does as a consequence of the dynamics of the interactions. Chew and Frautschi of Univ. of California refer to this situation as "bootstrap dynamics". No one has succeeded in calculating the properties of these particles as inevitable consequences of their strong interactions with each other, but it presents a fascinating challenge.

Four classification schemes, given the existing particles, have called for certain other particles. One of these schemes associates particles differing only in spin and mass, in analogy with an atomic system where each new A.M. value is associated with a particular quantised state. According to this scheme, any known particle should have more massive counterparts with the same values of b, I and Y, but with spins differing by two units. The lines linking such particles on a graph of spin versus mass are known as Regge trajectories. Only three pairs of particles linked by these trajectories are now known, but others are being sought.

Another classification scheme, due to Gell-Mann and Ne'eman, emphasises the fact that differences of mass between baryons is small compared with their mass. If we were to assume that all the spin 1/2 baryons are basically the same material, we might account for their mass separation in ~~response to~~ terms of their response to some interaction which arranges them in different states. Perhaps there is a parameter which responds to some part of the strong nuclear interaction to form supermultiplets. Just as the isotopic spin multiplets are different in electric charge, perhaps the supermultiplets could be different in hypercharge. We could group together particles having the same spin and parity, but differing in charge and hypercharge. Starting with the nucleon, with hypercharge Y = +1, a supermultiplet of this type would contain particles with Y = 0 & Y = −1. Our description of such a situation must contain terms accounting for the transformation of the system from one state to another. The transformations could occur with a change of charge, hypercharge or both together.



Y, I
−1, 1/2
0 1
+1 1/2



Y-Ichiway arrangement of some particles (b) Meson octet (Sc. Amer. 64)

Fig 71(a) gives a eight-fold arrangement of 8 B baryons ($S=\frac{1}{2}$, parity +ve) (excluding Ω^-) forming a super multiplet and also the connections are shown (what connections?). It is represented as a plot of hyperchargé versus charge, but it is also roughly a plot of mass versus charge. Transformation in Charge along the horizontal axis makes a difference of only 1% in mass (∵ Σ's & N's having little diffce in mass among themselves), but the mass differences produced by transformation of hypercharge are about 10 times larger (for eg. $1318 - 1193 = 125 = 10 \times 12.5 \simeq 10\%$ of $1250 \left\{ \simeq 1318 \text{ or } 1193 \right.$)

The description of these relationships in terms of the Lie algebra of groups (ie continuous groups like the L.T) has been very successful. This algebra specifies the number of interactions possible within a group having a particular number of members and transformation properties. In particular, it allows predictions to be made about the mass differences of these multiplets, and about the number of members to be expected within each super multiplet. In the case of the baryons of spin ½, the algebra calls for eight members as shown in fig. 71(a), including the two neutral particles with $Y = 0$. These two differ in total I. The predicted mass relationships are simple and agree very accurately with the measured values. For this particular group

$$\frac{1}{2}\left(m_N + m_\Xi\right) = \frac{3}{4} m_\Lambda + \frac{1}{4} m_\Sigma$$

The same octet pattern should be found for other strongly interacting particles with the same values of charge and hypercharge. The meson group of π, K and η shown in fig. 71(b) satisfies the relationship among masses provided that the squares of the masses are used [does this mean $\frac{1}{2}\left(m_K^2 + m_K^2\right) = \frac{3}{4} m_\eta^2 + \frac{1}{4} m_\pi^2$?]. This difference in the mass rule between mesons and baryons may be linked with the fact that the separation of masses for the mesons is as large as the masses themselves. The heavier and less stable resonance particles may also fall into similar patterns as shown in fig 72. but the actual values of spin and parity of many of these resonance baryons have not been measured.

Fig. 72 of 8-fold way for resonance particles, showing what they are (3C. Rev. 64)

The predicted value for the η^0 mass is 925 MeV. Indeed there are the η^0 with $Y = 0$, $I = 0$. The eightfold way can accommodate this situation.

[What does this mean?]

Octets are not the only arrangement possible. The same relationships between I and Y can also produce singlets and decuplets. In a decuplet of baryons, the mass differences between members of adjacent hypercharge should be equal. The arrangement is shown in Fig. 73. In this scheme, the first resonances to be discovered (3/2 spin, 3/2 I proton, pion resonances) forms the base. Examples of other particles in the decuplet were known in 1963 to allow predictions of the mass and other parameters of the missing members. In particular there should be a singlet with Y = −2, and mass 1676 Mev, which would be stable against strong nuclear decay. This is the Ω^- which was discovered in 1964.

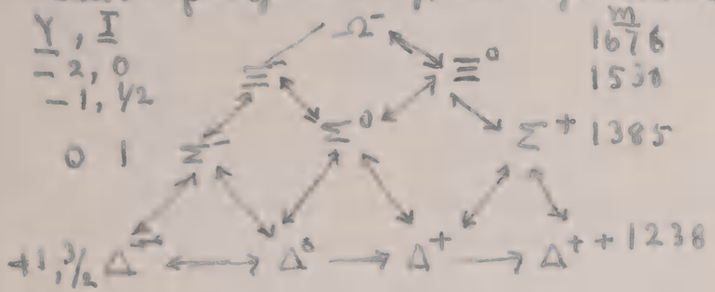


Fig 73 (Sc. Amer, 64)

The particular algebra which permitted these predictions is known as SU(3). The symbols stand for Special Unitary group for 3×3 arrays. The special condition eliminates one of the nine components, but leaving only 8 independent ones. Three of these are associated with isotopic spin orientations, one with hypercharge, two with transformations between states with adjacent hypercharge, and two with transformations between states with adjacent hypercharge and electric charge. The system has been known as the Eight Fold Way.

There may be other arrangements which encompass all of the particles and include the present organisational schemes as special cases. One possibility is that all the strongly interacting particles are manifestations of three more primitive forms. These have been called the "Quark". Among other strange features, they would have fractional electric charge.

Since the 1950's, the parameters of isotopic spin, hypercharge, and parity have been measured and assigned to the particles, but more important have been the changes of attitude towards these assignments. The organisation schemes are now more concerned with the relationships among particles than with the particles themselves.

Some of the problems, perhaps the crucial ones, will not be solved until we use the next generation of accelerators which have energies 10 times those now available. Hopefully, we will find that the weak interactions can indeed be explained in terms of one

me Weak interaction, and that the strong nuclear force (or forces) can account for its multiplet manifestations of a very small number of primitive particles. Perhaps, it can account for all the strongly interacting particles without any one of them being more primitive than the others. We may find that the W-particle (or particles) can account for the heavy existence of the heavy electron known as the muon. And more surely yet, we may expect that new problems will arise whose fundamental nature we cannot even now surmise.

Appendices (1) High energy accelerators — descriptions in detail

(2) Particle detection — highly technical

(3) Bibliography

(4) Tables and interactions

(5) Problems.

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(4) (i) The interactions

Gravity — Gravitational "charge" is mass.

" force between particles negligible.

Force falls off with $1/r^2$, velocity independent, always attractive.

Graviton, agent of force, not detected.

Strong nuclear — Short range.

Charge independent.

Strength when nucleons touch is now 100 times greater than electric force.

e-m — Charge (Q) quantized either + or −.

Agent of force is photons;

responsible for atomic & molecular binding & hence for most "forces" of everyday world.

Velocity dependent, changing aspect from electrostatic to electromagnetic dependent on relative velocity of source and observer.

force attractive or repulsive.

Weak interaction — 10^{-13} times weaker than strong nuclear

Responsible for β-decay radioactivity and particle decays taking longer than 10^{-15} sec.

(ii) The Rules

The describing all interactions

is independent of	leading to conservation of
space translation	momentum
time translation	energy - mass
space rotation	angular momentum
Zero of electric potential	charge
Inversion of space and charge together	Product of space parity and charge reflection
Reversal of time	Time parity
?	Baryons and leptons.

The strong and E.M (but not the weak) interactions

are independent of	leading to conservation of
Reflection of space	parity
" of charge	charge parity : I_3 and S.

The strong (but not the E.M & weak) interaction

is independent of	leading to conservation of
charge	Isotopic spin I

(iii) The parameters.

Spin (S)

In a magnetic field, a particle with spin $\frac{1}{2}$ can exist in $(2S+1)$ energy states

Isotopic spin (I)

Interaction with the E-M field separates baryons with isotopic spin I into $(2I+1)$ charged states

Parity, even or odd

The function describing a particle system remains unchanged, except for a possible change of sign, if the signs of all spatial coordinates is changed (space reflection). The function has odd parity if it changes sign, even if it does not.

Strangeness (S)

The charge centres of isotopic spin multiplets within the same class are not the same. The "strangeness" number signifies the amount of this displacement. The charge centres for the two classes are chosen to be those for pions and nucleons.

Baryon and lepton number

b $\quad$ l

The baryons have $b = +1$ for particles, and $b = -1$ for antiparticles.

The leptons have $l = +1$ for particles, and $l = -1$ for antiparticles

For baryons and mesons electric charge $q = \frac{1}{0}$ electron charge

$$q = e\left|I_3 + \frac{b}{2} + \frac{s}{2}\right|$$

Hyper charge

Y

Y equals twice the average charge of a multiplet

$$Y = s + b.$$

Constants and useful relationships

Velocity of light (c) $= 2.998 \times 10^8$ m/sec

Planck's const. (h) $= 6.625 \times 10^{-34}$ j. sec

Avagadro number = mole = N $\quad 6.025 \times 10^{23}$

Quantized charge unit e $\quad 1.602 \times 10^{-19}$ Coul.

Electron rest mass m_e $\quad 9.108 \times 10^{-31}$ kg $= 0.511$ MeV

Proton rest mass m_p $\quad 1.672 \times 10^{-27}$ kg $= 938$ MeV

Boltzmann's const. K $\quad 1.38 \times 10^{-23}$ j/°K $= 8.62 \times 10^{-5}$ eV/°K

Electron volt eV $\quad 1$ eV $= 1.6 \times 10^{-19}$ joule

Magnetic field: 1 weber/m^2 $= 10^4$ gauss

Horizontal comp. of earth's field in Nevis γ-ville $= \frac{1}{5}$ gauss

Large iron magnets saturate at ≈ 2 weber/m^2

Magnetic moment of electron $\mu_e = 5.79 \times 10^{-8}$ eV/gauss $= 5.79 \times 10^{-4}$ eV/weber/m^2

Momentum-wave length: $p = \frac{h}{\lambda}$

Total energy $= E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} = \frac{m_0 c^2}{\sqrt{1-\beta^2}} = m_0 c^2 + \frac{1}{2} m v^2 +$ higher terms in v/c.

$$E^2 = c^2 p^2 + (m_0 c^2)^2$$

Time-dilatation $\Delta t = \Delta t_0 / \sqrt{1-\beta^2}$

Heisenbergh uncertainty relationships: $\Delta x \cdot \Delta p_x \geq h \approx 1 \times 10^{-34}$ j. sec

$\Delta\theta \cdot \Delta L \geq h \approx 7 \times 10^{-16}$ eV. sec

$\Delta t \cdot \Delta E \geq h \approx 7 \times 10^{-22}$ MeV. sec

Exponential decay : $\Delta N = \lambda N \Delta T$, $N = N_0 e^{-\lambda t}$

$$T_{\text{mean life}} = \frac{1}{\lambda} = 1.45\, T_{1/2 \text{ life}} \; ; \; T_{1/2} = \frac{0.69}{\lambda}$$

Photon energy and momentum : $E = h\nu$

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

Compton effect wave length change : $\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$

In usual scattering, m_0 is electron mass :

$$\lambda' - \lambda = 2.43 \times 10^{-12} \text{ meter } (1 - \cos\theta) .$$
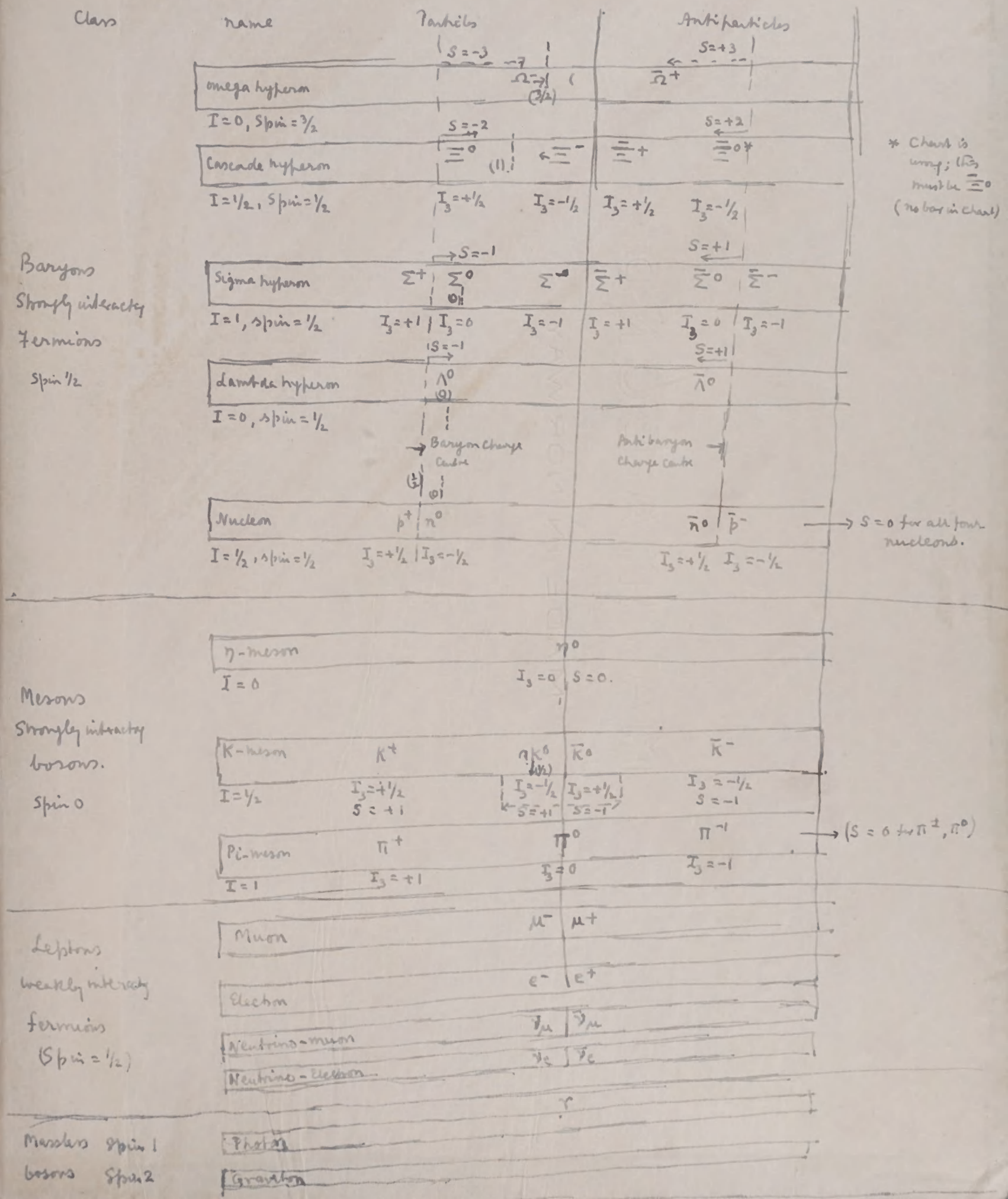
Chart (from Swarup) [left side only] p.32.